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Divergence and Convergence in Scarf Cycle Environments: Experiments and Predictability in the Dynamics of General Equilibrium Systems

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SOCIAL SCIENCE WORKING PAPER 1239

Forthcoming Economic Theory

April 2020

ABSTRACT
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ABSTRACT

Previous experimental work demonstrates the power of classical theories of economic dynamics to accurately characterize equilibration in multiple market systems. Building on the literature, this study examines the behavior of experimental continuous double auction markets in convergence-challenging environments identified by Scarf (1960) and Hirota (1981). The experiments provide insight into two important economic questions: (a) do markets necessarily converge to a unique interior equilibrium? and (b) which model, among a set of classical specifications, most accurately characterizes observed price dynamics? We observe excess demand driven prices spiraling outwardly away from the interior equilibrium prices as predicted by the theory of disequilibrium price dynamics. We estimate a structural model establishing that partial equilibrium dynamics characterize price changes even in an unstable general equilibrium environment. We observe linkages between excess demand in one market and price changes in another market but the sign of expected price change in a market does not depend on the magnitude of excess demand in other markets unless disequilibrium is severe.

[1] The financial support of the John Templeton Foundation the Rising Tide Foundation and the Caltech Laboratory for Experimental Economics and Political Science are gratefully acknowledged. Comments of Peter Bossaerts, Anjan Mukherji and Bill Zame contributed significantly.

1. Introduction

The paper studies and experimentally confirms the ability of classical economic principles of multi-market dynamics to predict spiraling paths and exploding price dynamics. The possible existence of such complex market behaviors is predicted by the theoretical works of Scarf (1960) and Hirota (1981). The study adds to existing support of basic principles of multi-market dynamics and the use of laboratory experimental economics methods to study special features of phenomena that are otherwise impossible to study in complex, naturally occurring economies. Previous experimental work demonstrates robust convergence of prices to the competitive equilibrium, possibly punctuated by aspects of local instability and bubble-like price patterns. Many of these studies explored theoretically stable environments in which the predictions of equilibrium analysis and multi-market price dynamics stand broadly in accord. Predictions of price divergence to a boundary and the possibility of perpetual price movement suggested by Scarf and Hirota's model of price dynamics, which stand in sharp contrast to the classical predictions of equilibrium, had not yet been explored in the laboratory setting. In so doing, the results identify the underlying principles driving price dynamics as devices that underscore the value of general equilibrium theory for understanding market behavior.

The Scarf (1960) and Hirota (1981) models consider price formation from the abstract perspective of a Walrasian auctioneer capable of measuring excess demand at disequilibrium prices without implementing a market system in which trades take place at such prices. While this abstraction is well-suited to forming logical conceptualizations of market adjustment processes, it also presents a major departure from the procedural details of how actual markets function and the strategic behavior of individuals participating in markets. Experimental markets, by contrast, implement actual trading institutions and features of price discovery in which there is no fictional Walrasian auctioneer. Instead, bids and asks are tendered by potential traders themselves in real time and, as trades take place at different prices, demand and supply curves shift as trading proceeds. Vernon Smith (1965)'s discovery of price convergence in market experiments demonstrates the close connection between a theory derived from abstractions and the data drawn from a completely different environment. The abstractly formulated theory of price processes generates predictive power even when applied to a very different environment subjected to a host of frictions assumed away by the theoretical abstraction. Many studies of experimental markets show that markets tend to "equilibrate" at a pattern of prices and allocations that are near the equilibrium of the fictional Walrasian auctioneer.[2]

[2] The role abstract, axiomatic principles of (multiple) market behavior which address the details of neither market institutions nor individual decisions, have been a topic of much discussion in the economics literature. The experimental markets converge to the equilibrium predictions of a model that clearly lacks

Previous experiments lend support to the principles of equilibrium and the role of excess demand underlying dynamic price formation in the classical competitive model. Researchers have demonstrated the continuous double auction's convergence toward the competitive equilibrium prices and allocation is a reliable property of markets under a wide range of environments (Plott and Smith, 2008). In specialized conditions, though, unstable equilibria, price bubbles, and cyclic price movements have all been observed and studied as features of price movements. This paper explores these possibilities in a novel setting with previously untested predictions. By implementing an experimental market featuring extensive disequilibrium trading, we further exploit the information from observing this trading to better understand subtle features of price adjustment.

In light of the deep experimental literature exploring the price adjustment mechanism, we relate our current study to key contributions from this literature in Sect. 2. The Scarf (1960) and Hirota (1981) models we study present an informative theoretical framework for exploring market price dynamics. Applying theory to the specification we test identifies the possibility of explosive prices, or the possibility that the Walrasian auctioneer's algorithm for price discovery fails to converge, not yet demonstrated in experimental markets. Section 3 presents this theoretical model, characterizing market participants' preferences, endowments, and the resulting equilibrium. We also characterize excess demand in this market under classical Tatonnement, which suggests that a price discovery mechanism may settle into a limit cycle following a very slowly expanding orbit featuring extensive disequilibrium trading.

Section 4 introduces the experiment we use to explore these theoretical predictions, detailing many of the practical elements of the implemented market design. The experimental setting, itself, builds on a host of developments in the design and implementation of experimental markets. This discussion provides background on the experimental foundations of the design to complement the theoretical basis of the study, reviewing the classical MUDA market and detailing institutional features specific to its current implementation.

Given our interest in understanding the price formation process, Sect. 5 discusses the observed price dynamics, presenting suggestive evidence in support of the key model prediction that prices fail to converge to the theoretical equilibrium. Observed prices spiral around the equilibrium as predicted by models of excess demand before eventually hitting a price boundary, well away from equilibrium. Statistical tests demonstrate that

descriptive accuracy. While the markets operate "as if" the Walrasian auctioneer is active and in control, the precise reasons for the accuracy of the model is, as Vernon Smith asserts, a mystery.

(a) prices trend away from the theoretical equilibrium, (b) the random price changes are only weakly attracted to theoretical equilibrium.

Section 6 explores the market adjustment process from the perspective of excess demand and allocation efficiency. Here, we note that excess demands do not converge to zero, and gains from trade persist despite the cessation of trading activity. These gains from trade, however, can only be realized through complex, multilateral trades. In fact, our experiments realize a high level of allocative efficiency for complex markets, realizing 75% of potential gains from trading relative to equilibrium and 94% of potential gains from trading relative to a random reallocation of underutilized commodities.

Though the price dynamics and persistent excess demands aren't compatible with static equilibrium analysis, they are consistent with predicted dynamics implied by classical models of price formation. Further, the substantial trading activity that occurs at disequilibrium prices provide a valuable opportunity to identify the forces that drive price dynamics. This data allows us to build on previous investigations of price formation by estimating structural models relating the underlying parameters governing excess demand to price dynamics in Sect. 7. Our analysis not only evaluates the relative significance of excess demand and prices as drivers of observed prices, but also gives insight into the relative importance of partial equilibrium forces and excess demand spillovers between markets in determining prices and allocations.

Our results show that classical models of Tatonnement generate powerful predictions of price formation, and that partial equilibrium analysis is largely sufficient to characterize price dynamics. While market linkages as part of general equilibrium forces are non-negligible, we find price changes for a good are largely determined by excess demand and disequilibrium in its own market with only second-order influences of disequilibrium in other markets. These results support a fundamental principle from Walras and others that the direction of price change in a given market depends only on the sign of its own excess demand. We only observe this principle being violated under conditions of extreme disequilibrium, possibly through expectations of future prices in interdependent markets.

2. Literature Review

The literature on market stability opens a window for viewing relationships between excess demands and disequilibrium behavior in markets, highlighting substantial differences in the mechanisms by which prices react to imbalances in excess demand. Leon Walras advocated a market adjustment theory that price responds to a difference between quantity demanded and quantity supplied at the price. The Marshallian perspective viewed quantity adjusting to profitability at the margin, so if demand price exceeded supply price at a given quantity, the quantity increased. In both Walras and Marshall, instability was viewed as emerging from a case of "perversely" sloped demand (upward sloping) or supply (backward bending or downward sloping). The "perverse" shapes of a Walrasian model were attributed to income effects while the "perverse" shapes of the Marshallian model and thus instability is associated with externalities, originally motivated by industry level economies of scale.

It was only after the refinements of theory and advances in laboratory experimental methods, over a hundred years later, that the underlying theoretical principles were observed in operation. The phenomenon of market instability was first observed in Plott and George (1992) based on a falling supply curve reflecting a Marshallian externality.[3] Instability due to an upward sloping demand created through Walrasian motivated income effects (e.g. inferior goods), is first observed in Plott (2000).

The classic papers by Scarf (1960) and Hirota (1981) introduce an environment in which excess demand driven markets need not converge to an interior equilibrium. Instead, excess demands directed a path for prices which could cycle around an interior equilibrium need not result in an equilibrium point. These models allowed experimental markets to focus on the time-series path of prices and not simply the stability or instability of an equilibrium point. Experiments conducted by Anderson, Granat, Plott and Shimumura (2004, henceforth AGPS) examined this environment with the double auction institution. They observed that prices moved close to an interior equilibrium when the models predicted convergence to an interior equilibrium. When the model predicted prices spiraling around the equilibrium the experiments produced spiraling price movements in the predicted direction but the hypothesis that the data were slowly moving towards the interior equilibrium could not be rejected. The Scarf conjecture that a multiple market system need not equilibrate at an interior equilibrium could neither be confirmed nor rejected even though major features of the conjecture were clearly confirmed. The AGPS results are replicated in simulations by Goeree and Lindsey (2016).

The power of excess demands to guide market price and allocation adjustments is explored further by Crocket, et.al. (2010) who studied the power of market dynamics when placed against the possible, resulting income distribution. A two goods market economy in which a stable Walrasian equilibrium with extreme income differences emerges when placed in competition with a price unstable allocation with an egalitarian distribution. The experiment illustrates the power of the market dynamics as compared to

[3] The instability results were replicated for an upward sloping demand by Plott and Smith (1999) through a (Marshallian) demand externality implemented as a fad in which demands of individuals are influenced by the consumption patterns of others.

models of equilibrium selection based on income distribution. Prices move toward the competitive equilibrium when predicted by the dynamics even though the incomes of some agents are driven to low levels.

Gintis (2007) presents simulations demonstrating equilibration in a Scarf-type environment when prices are only privately known and thus uncoordinated. Indeed, Gintis (2007) conjectures that public prices such as competitive prices or the prices in the double oral auction are a coordinating device that creates the instability of the Scarf (1960) and Hirota (1981) models. Shen, Shimomura, Yamato, Ohtaka and Takahashi (forthcoming) test the Gintis idea using the three equilibrium design of Plott and George (1992) in which three equilibria are created with income effect driven demand that differ according to Walras and Marshallian stability. Rather than the double auction with transparent public order flow, the Shen, et.al. (forthcoming) market was organized as a "pit market" in which communication of bids and offers is confined to the pair of interacting traders. In this environment, predictions based on Marshallian principles best explain the data in contrast to the results of Plott and George (1992). Why the organizational change had such a profound effect is an open question but the changes of information available is a possible explanation. The Marshallian externality and associated instability demonstrate that "other" market activities that have the capacity to change preferences in a given market, introducing a systematic influence on stability. That interdependence could extend to information flows. For example Plott and Pogorelskiy (2017) demonstrate that information about order flow influences convergence. Thus, information related to other markets, discussed later as forms of information spillovers among markets and market linkages, could have a systematic influence on stability.

The setting we study here is closest to that of AGPS, as the preferences are derived within the same theoretical framework, market organization and experimental procedures considered by AGPS, though we study a substantially different set of parameters with distinct predictions about market behavior. While the AGPS treatment predicted convergence to equilibrium or a limit cycle, the specification we study allows for potentially explosive price dynamics (subject to budget constraints). This setting allows us to demonstrate that, consistent with the predictions derived by excess demand, observed prices move away from the interior equilibrium. Since the parameters used here predict extreme dynamics, the experiments studied here can also be interpreted as giving support to the results of Crocket, et.al. (2010) that markets dynamics are driven by excess demands as opposed to income distribution.

We further extend the analysis presented in AGPS to study the structural equations relating price changes to excess demands. Doing so allows us to test the possibility of market linkage or spillovers in the effect of excess demand in one market on price changes in other markets. Though we do find evidence of such linkages between markets, those linkages are weak and present second-order influence on price behavior in a market relative to excess demand for its own good. While this evidence is presented within the specific market context we study here, we frame our findings in a general structural model based on economic theory that can be extended to any market setting. This analytical framework invites exploration of the degree to which these findings extend to other settings and market specifications.

3. Theoretical Model for Preferences, Equilibrium, and Price Adjustment

Our experiments induced subjects' preferences to be similar to those studied theoretically by Scarf (1960) and Hirota (1981) and experimentally by Anderson, et.al. (2004), and Plott (2001), albeit with substantial modifications. Agents have perfectly complementary Leontief preferences for two of the commodities while deriving no utility from the third. "Appendix 1" presents the general form of these models and relates this general form to the current implementation. By design, these preferences and initial endowments ensure the existence of a unique, interior competitive equilibrium for all experiments. However, the existence of this equilibrium need not imply observed transactions occur exclusively at equilibrium prices.

3.1. Commodities, Preferences, and Endowments

We implement two markets for the commodities X_1 and X_2 featuring continuous double auctions with a limit-order book where prices of a single unit of X_1 and X_2 are quoted in terms of the number of units X_3. This implementation places commodity X_3 as the numeraire, allowing us to plot prices of commodities X_1 and X_2 in terms of units of X_3.[4]

We selected preferences and initial endowments so a classical model of price adjustment predicts prices will diverge from the competitive equilibrium given any non-equilibrium initial price vector and do so in a predictable fashion. In the next subsection, we demonstrate how the unique, interior, competitive equilibrium is unstable according to this classical model. The nature of this instability depends on preferences and endowments, which we explore in two opposing specifications, the "Clockwise" and "Counterclockwise" treatments.[5] Table 1 presents the specific magnitudes of the utility

[4] In all of our specifications, endowments of commodity X_3 are much greater than the endowments of the other two commodities. Since prices are in terms of units of X_3, finely divisible units of X_3 must exist to prevent the integer constraint from substantially reducing the number of feasible prices.

[5] Previewing results from the next section, the classical model predicts prices under the clockwise (counterclockwise) treatment will diverge in a clockwise (counterclockwise) direction when plotted in the two-dimensional price space for Commodities X_1 and X_2, treating X_3 as the numeraire.

parameters and initial endowments, with Fig. 1 illustrating indifference curves for each type in the Clockwise treatment.

Table 1: Preferences, Endowments, and Equilibrium Allocations (X_{1i}, X_{2i}, X_{3i})

Clockwise	Type 1	Type 2	Type 3
Utility	$70 \min\left\{\frac{3X_{2i}}{4}, \frac{X_{3i}}{80}\right\}$	$70 \min\left\{\frac{X_{1i}}{2}, \frac{3X_{3i}}{80}\right\}$	$70 \min\left\{\frac{3X_{1i}}{2}, \frac{X_{2i}}{4}\right\}$
Endowment	(0, 0, 800)	(20, 0, 0)	(0, 40, 0)
Equilibrium	(0, 10, 600)	(15, 0, 200)	(5, 30, 0)
Counterclockwise			
Utility	$210 \min\left\{\frac{X_{2i}}{12}, \frac{X_{3i}}{80}\right\}$	$210 \min\left\{\frac{X_{1i}}{2}, \frac{X_{3i}}{240}\right\}$	$210 \min\left\{\frac{X_{1i}}{6}, \frac{X_{2i}}{4}\right\}$
Endowment	(0, 40, 0)	(0, 0, 800)	(20, 0, 0)
Equilibrium	(0, 30, 200)	(5, 0, 600)	(15, 10, 0)

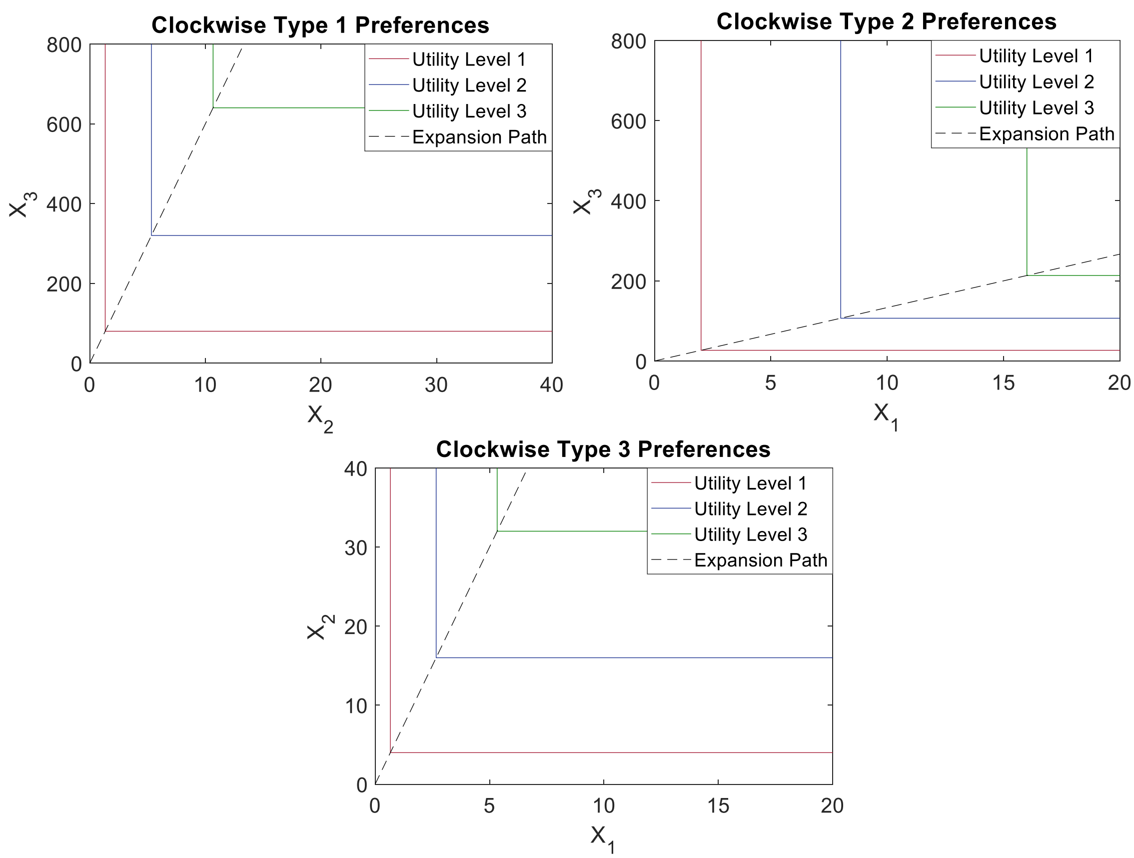

Fig. 1: Indifference Curves for Each Clockwise Treatment Type

3.2. Competitive Equilibrium in the Classical Model and Excess-Demand Dynamics

In a setting with two commodities X_1 and X_2 and prices P_1 and P_2, denote the excess demands for each good by $Z_1(P_1, P_2)$ and $Z_2(P_1, P_2)$, respectively. The classical competitive equilibrium model defines the theoretical equilibrium to be the prices at which both $Z_1\left(P_1,P_2\right)=0$ and $Z_2\left(P_1,P_2\right)=0$. In a static setting, this equilibrium predicts final prices for trades and allocations based on a fixed demand and supply. In a dynamic context, these equations provide tools for measuring forces that underlie price discovery. They are the structural equations that connect price movements to the underlying parameter of the environment.

We define the "Classical Model" of dynamics evolving from Walras' fundamental principle that prices respond to excess demand in a good's own market. Under this model, the change in prices for goods X_1 and X_2 ($\dot{P}_1$ and $\dot{P}_2$, respectively) scale linearly with the excess demand for each respective good, so that

$$\dot{P}_1 = a_{11} Z_1\left(P_1,P_2\right), \text{ and, } \dot{P}_2 = a_{22} Z_2\left(P_1,P_2\right). \tag{1}$$

The parameters a_{11} and a_{22} reflect the relative speed with which a market price accommodates, or adjusts to, its excess demand. These parameters can play a central role in characterizing models of dynamics and stability. Hicks, for example, develops a model in which markets adjust at different rates as central feature of his model of partial and general equilibration.[6]

Table 2: Market Excess Demand for Commodities X_1 and X_2 at Initial Endowments

	$Z_1\left(P_1,P_2\right)$	$Z_2\left(P_1,P_2\right)$
Clockwise	$\frac{60P_1}{40+3P_1}+\frac{40P_2}{P_1+6P_2}-20$	$\frac{800}{60+P_2}+\frac{240P_2}{P_1+6P_2}-40$
Counterclockwise	$\frac{800}{120+P_1}+\frac{60P_1}{3P_1+2P_2}-20$	$\frac{40P_1}{3P_1+2P_2}+\frac{120P_2}{20+3P_2}-40$

Given agents' preferences and endowments, we can solve for the excess demand equations in the exchange economy.[7] Letting $P = (P_1, P_2)'$ denote the prices for X_1 and X_2 with X_3 as a numeraire, Table 2 presents the market excess demand function at initial endowments when each type of agent is present in equal proportion. Setting the

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[6] Hicks posed a question about the relationship between partial equilibrium and stability. McFadden (1969) formalizes a concept of partial equilibrium demonstrates a close connection between Hicksian conditions for partial equilibrium and Samuelsonian models of stability. One could imagine alternative processes for price dynamics and a substantial literature explores the properties of such alternatives. We will discuss such alternatives later, along with useful empirical generalizations, in the next subsection.

[7] Assuming perfectly liquid markets in which agents' behave as price-takers allow us to specify their demand (or supply) of each commodity considering only their budget constraint and initial endowments. Summing these individual demand functions and subtracting the total endowments of each commodity characterizes the market excess demand function. Equilibrium then attains when excess demand functions equal zero and market demand equals market supply for each commodity.

equations in Table 2 to zero yields the theoretical equilibrium prices for X_1 and X_2 equal to 40 and 20 in both the clockwise and counterclockwise treatments. The implied allocations under this equilibrium appear alongside the endowments in Table 1.

This experimental design is inspired by the theoretical literature on classical principles of dynamic adjustment as discussed in the next subsection.[8] The market specifications from Scarf (1960) and Hirota (1981) provide settings in which classical forces do not guide prices to converge toward the competitive equilibrium. As presented in Fig. 2, a simple phase diagram characterizes the dynamic behavior of the "Classical Model" in which $\dot{P}_1 \propto Z_1(P_1, P_2)$ and $\dot{P}_2 \propto Z_2(P_1, P_2)$. The partial equilibrium curve $Z_1(P_1, P_2) = 0$ represents the pairs of prices for which the excess demand of X_1 is zero regardless of excess demand for X_2, with the curve $Z_2(P_1, P_2) = 0$ defined analogously. These curves intersect at the theoretical equilibrium prices $P^* = [40, 20]$, with zero excess demand for both goods.

The partial equilibrium curve for commodity X_1 divides the price space into regions in which the excess demand for X_1 is positive (negative), placing upward (downward) price pressure on X_1. The partial equilibrium curve for commodity X_2 similarly divides the space into areas in which the excess demand for X_2 is positive (negative) so the price pressure on X_2 is up (down) according to the theory. The price space is thus partitioned into four regions in which simple excess-demand driven models of dynamics make definite predictions for the direction of price movements. The implied direction of these movements is shown by the small arrows in Fig. 2, with the directed lines presenting a simulated price path based on Eq. (1) and a given initial position.

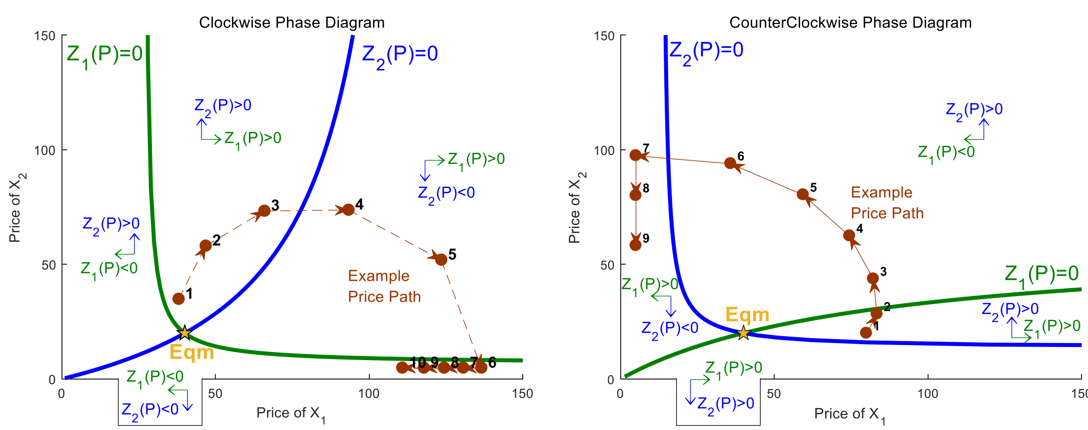

Fig. 2: Excess Demand Phase Diagrams for Simple Dynamic Model

[8] Though our market implementation deviates from the frictionless assumptions imposed by the price adjustment processes derived in much of this literature, the design is driven by predictions from these classical models of dynamics.

From the initial position, the classical model's simple adjustment process predicts price dynamics and possible non-convergence in both treatments. Contrasting Fig. 2 for the Clockwise and Counterclockwise specifications identifies the difference in predicted price dynamics for the two treatments. In the Clockwise treatment, classical dynamics suggest prices move in a clockwise path around the competitive equilibrium, so the angle of prices relative to equilibrium (P_1, P_2) plane declines as prices adjust (until it jumps upon passing zero). In contrast, the same model predicts prices in the Counterclockwise treatment move in the opposite direction, counterclockwise around equilibrium prices.[9] This contrast in predictions in the two treatments under the classical model, coupled with the expectation of substantial disequilibrium trading activity due to the potential for non-convergence, present a powerful empirical setting in which to explore the dynamic response of prices to excess demand in a multi-market setting.

Note that the example price paths depicted in Fig. 2 only represent *expected* changes in prices. The actual price paths will be affected by substantial unmodeled variability, including behavioral artifacts, microstructure noise, and misspecification, leading to transactions executing at a wide variety of prices. The substantial unpredictable component of price dynamics is to be expected in light of market forces limiting potential arbitrage opportunities. Separating this signal from the noise requires econometric evaluation to resolve the empirical question of whether predictions from excess demand models effectively characterize expected price dynamics.

3.3. The Generalized Classical Model and Structural Models for Disequilibrium Price Dynamics

The theoretical discussion in the previous subsection presents the "Classical Model" of disequilibrium price dynamics in which the rate at which prices change are proportional to their excess demand. In modeling the joint price process for multiple goods, the dynamics from Eq. (1) provide a structural model of price formation by the difference equation:

$$\begin{bmatrix} \dot{P}_{1,t} \\ \dot{P}_{2,t} \end{bmatrix} = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{pmatrix} Z_1\left(P_{t-1}\right) \\ Z_2\left(P_{t-1}\right) \end{pmatrix}.$$

A simple generalization embeds the Classical Model as a special case of the "Generalized Classical Model,"

$$\begin{aligned} \dot{P}_1 &= a_{11} Z_1\left(P_1, P_2\right) + a_{12} Z_2\left(P_1, P_2\right) \\ \dot{P}_2 &= a_{21} Z_2\left(P_1, P_2\right) + a_{22} Z_2\left(P_1, P_2\right), \end{aligned} \tag{2}$$

[9] In these figures, as in experimental sessions, we impose a floor on both P_1 and P_2 equal to 5 units of X_3 to avoid technical issues from trades at zero-prices. Also, while certain regions of these diagrams suggest explosive price dynamics, we note that the limited supply of X_3 imposes an effective ceiling on P_1 and P_2.

The Generalized Classical Model allows for price adjustments of markets linked by a "linked adjustments principle" or "excess demand spillovers." That is, the adjustment in a single market depends on the state of disequilibrium (as measured by excess demand) in other markets. The classical model is the special case that satisfies the restrictions $a_{11} > 0,\ a_{22} > 0, \text{and},\ a_{12} = a_{21} = 0$.

We originally explored the generalization as a technical tool to evaluate the magnitude of deviations from the Classical Model in which the off diagonal elements are zero. However, we discovered the generalization plays a deeper role in the theory of dynamic equilibration. Introducing possible sensitivity for price adjustments to the degree of disequilibrium in other markets, measured by the size of the excess demands, provides insight into the possible role of uncertainty in disequilibrium dynamics. While supply might be greater than demand for commodity X_1, the disequilibrium in the market for X_2 might attenuate the rate at which P_1 decreases or even cause P_1 to increase rather than decrease. Walras and others tended to reject this as a possibility and postulated the "fundamental principle" that the direction of price change of a given commodity depends only on the sign of its own excess demand.[10]

Excellent reviews of classical price dynamics are presented by McKenzie (2002) and by Mukherji (2002, 2003, 2019). The models presented by Eqs. (1) and (2) present special cases of theories that have the following form:

$$\dot{P} = A(P)Z(P)$$

Here, $\dot{P}$ represents the change in prices over time, P is the price vector. We refer to $A(P)$ as an adjustment matrix of coefficients that may depend on prices P, and $Z(P)$ is a vector of excess demands as a function of prices. Though the theory generalizes to any number of commodities, we focus on the two-dimensional price setting implemented here for ease of exposition.

The primary feature of this model is that price changes depend upon P through the adjustment matrix $A(P)$ in addition to the functional relationship by which prices enter the excess demand functions. Though the dependence of the adjustment matrix on prices could take any form, we look to the literature to identify plausible restrictions that impose

[10] Walras (1954, p.85) states three suppositions that collectively state that the sign of the excess demand and the sign of price changes will be the same. As mentioned above, Hicks appears willing to postulate the existence of a linkage. As will be mentioned below, Edgeworth presents a different opinion based on a different model of price adjustments.

some structure on this relationship.[11] In particular, we explore specifications of the Classical and Generalized Classical models where the elements of $A(P)$ vary with P so as to characterize relative price dynamics. Specifically, consider a "Generalized Relative Model" in which price dynamics take the following form:

$$\begin{pmatrix} \dot{P}_1 \\ \dot{P}_2 \end{pmatrix} = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{pmatrix} Z_1(P) \\ Z_2(P) \end{pmatrix} \tag{3}$$

In the Generalized Relative Model, the function $A(P)$ decomposes into a matrix that contains the prices and a matrix of constants which pre-multiplies the excess demand functions. This model of price dynamics can be equivalently expressed so that the percentage change in price depends on excess demands through a constant adjustment matrix. To illustrate in the two commodity case for any single commodity, i, the adjustment process in Eq. (3) can be written as:

$$\frac{\dot{P}_i}{P_i} = a_{i1} Z_1(P) + a_{i2} Z_2(P), \; i = 1, 2.$$

Of course, this model can be further refined by hypotheses focused on the numbers a_{ij}. In particular, the Classical Relative Model presents a restricted case of Eq. (3) in which the off diagonal elements are restricted to be zero and the diagonal elements positive. Table 3 summarizes this section, consolidating the dynamic models we consider for ease of reference.

We use these predictive expectations as the foundation for a structural econometric analysis of the price formation process in Sect. 7. This analysis allows us to test classical restrictions on market linkages and evaluate the empirical significance of spillover effects of excess demand from one market to price formation in other markets.

Table 3: Predictive Expectations for Price Dynamics

Specification	*Absolute Model*	*Relative Model*
Classical	$\begin{bmatrix} \dot{P}_{1,t} \\ \dot{P}_{2,t} \end{bmatrix} = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{pmatrix} Z_1(P_{t-1}) \\ Z_2(P_{t-1}) \end{pmatrix}$	$\begin{bmatrix} \dot{P}_{1,t} / P_{1,t-1} \\ \dot{P}_{2,t} / P_{2,t-1} \end{bmatrix} = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{pmatrix} Z_1(P_{t-1}) \\ Z_2(P_{t-1}) \end{pmatrix}$
Generalized	$\begin{bmatrix} \dot{P}_{1,t} \\ \dot{P}_{2,t} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{pmatrix} Z_1(P_{t-1}) \\ Z_2(P_{t-1}) \end{pmatrix}$	$\begin{bmatrix} \dot{P}_{1,t} / P_{1,t-1} \\ \dot{P}_{2,t} / P_{2,t-1} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{pmatrix} Z_1(P_{t-1}) \\ Z_2(P_{t-1}) \end{pmatrix}$

[11] As we are unaware of any attempt to study this model in its most general form, we focus on classes of special cases, although the literature is rich with discussion about the conditions under which less information is required for convergence. See Mukherji (1995) for a summary of recent literature, and for a treatment of stability in three commodities (two prices) models see Mukherji (2004).

4. Market Environment and Experimental Design

Experimental methods in economics evolved as tools to create simple and special case markets in which the broad, abstract and general principles of economics can be studied under controlled conditions. The key elements are the commodity space, the preferences and the trading institutions, all of which support the creation of a simple market system to which general economic principles apply. While the experimental markets are simple special cases of markets, they are nevertheless, real markets. Though simplicity should not be confused with reality, general principles are expected to apply even in the simple and special cases.

The methods rest on the creation of a commodity space and the use of money to induce preferences over the commodities that can be traded. Time exists as a "period" in which trading of money and commodities take place in real time with the benefits of trading in terms of money earned are realized at the end of a period. The experiment proceeds as multiple periods or trading days that could be interpreted as a week. In a stationary environment, the periods are identical except for the information and benefits gained from previous periods. Trading takes place in a market organized by an architecture of institutions known to support efficient trading. Models are applied with an "as if" methodology with trading within a period and over periods both studied with equilibration predicted by models expected after multiple periods.

4.1. Market Architecture

Market creation required many operational decisions to ensure a feasible experimental protocol while preserving essential elements of the general equilibrium setting we seek to study. The markets were conducted as a continuous, multiple unit double auction, MUDA, introduced by Plott and Grey (1990) through an electronic market place developed by the Caltech Laboratory for Experimental Economics and Political Science (EEPS) called Marketscape. This market platform supports multiple, simultaneous, continuous markets. The markets are open for a fixed time called a period similar to a trading day.

When a period opens traders are free to tender bids to buy as a price per unit and maximum quantity or ask to sell units (also as a price per unit and maximum quantity). The markets have open (public) books that record bids and asks whenever they do not automatically trigger a trade by matching an already existing bid or ask in the order book. The bids are exposed to the market with price priority from highest price to lowest, while asks are exposed with price priority from lowest to highest, with ties prioritized by the time at which the bid or ask was entered. When a trade executes, the transaction is immediately recorded and units of inventory and money are instantaneously transferred between trading parties. Orders may be partially filled, with any unfilled portion remaining on the order book. Bids and asks remain in the book throughout a period unless expired, cancelled, or executed in a trade. In addition to the order book, all participants are able to view all data from all trades in continuous time through either a periodically updated graph or a listing of executed trades.

When a period closes subjects acquire the money they made, based on their end-of-period holdings according to induced preferences (described in Sect. 3.1), from the contracts they developed during the period. Upon the close of a period, the system validates accounting to record profits earned by participants based on the end-of-period holdings. Subjects' inventories are then reset to their initial endowments, the order book is cleared, and a new period of trading opens. Since stocks cannot be traded across periods, each trading period can be analyzed as a single market realization. However, the periods within a given session are not independent due to substantial price persistence from the end of one period to the beginning of the next.

4.2. Procedures and Experimental Design

Six separate experiments were conducted, all at the California Institute of Technology in the Laboratory for Experimental Economics and Political Science (EEPS) between November 2002 and July 2003. Each experiment consisted of a number of subjects modulo 3, as we require that there be an equal number of subjects of each type. The actual number of subjects in the experiments ranged from 9 to 18, with Table 4 summarizing the sessions conducted using each treatment specification. Participants included Caltech undergraduate and graduate students, as well students from Pasadena City College, many of whom were familiar with the software from previous (unrelated) experiments, but who did not necessarily have any training in economics.

Types were assigned sequentially to subjects as they logged into the software, and the order in which this occurred was essentially random. Subject payments averaged about $40.00 per subject per experiment. Experiments lasted no more than three hours. Upon arrival in the laboratory, subjects were given written instructions; including both a numeric table and a graphical display of indifference curves that represented their own (and only their own) endowments and induced preferences. In addition, we included an unrelated payoff table that illustrated how to read their true payoff table (which differed across subjects).

Each experiment began with a practice trading period serving several purposes. It acquainted subjects with the computers and software, so that they were comfortable with how to execute bid and ask offers before the paid portion of the experiment began. It also allowed time for the subjects to study their payoff information. Finally, it worked as a device for influencing initial conditions as we requested all trades in the practice period take place at a price of 25 units of X_3,[12] essentially providing a focal point for prices ahead of the first actual period. Thus, the initial starting point would be (25,25).

Following the practice period, each experiment consisted of a number of trading periods, ranging from 9 to 19 periods per session. Each period, in turn, lasted between 8 and 18 minutes dictated by the number of subjects and associated volume.[13] To avoid any "last period" effects, the final period was not announced as such until *after* it had concluded. After the close of the final period, earnings in francs were tallied and converted to dollars via a conversion factor. Subjects were then either paid in cash before they left the laboratory, or else checks were mailed to them shortly thereafter.

Table 4: Experimental Sessions

Treatment	**Date**	**Periods**	**N**	**Experienced Included**
I. Clockwise	11/27/2002	10	18	No
	12/11/2002	14	12	No
	7/17/2003	11	18	Yes
II. Counterclockwise	1/30/2003	12	15	Yes
	4/28/2003	9	15	Yes
	6/20/2003	19	9	Yes

[12] We did not establish this focal point for prices in the first session, experiment 021127. We note that these starting prices differ from the initial points in Fig. 2, which were chosen specifically to illustrate the cyclical features of the model. We discuss the manifestation of these cyclical dynamics from the training prices, including the role of integer constraints and unmodeled variability in prices, in detail in Appendix 2.

[13] Other issues caused time differences. These include training, software problems, subject mistakes and typos that required attention, differences in subject arrival times, etc.

5. Experimental Price Dynamics

We note that the continuous double auction institution involves trading executed at disequilibrium prices and, over the course of each period, at a variety of such prices. The markets produce two different time series of contract prices. First, "instantaneous prices" consist of the contracts that take place within a period. In experiments with one commodity instantaneous prices within a period typically exhibit erratic movement towards the "competitive equilibrium price."[14] Second, "period prices" record the evolution of prices across periods as summarized by the quantity-weighted average price within a period. Findings from other experiments suggest (1) instantaneous prices tend to converge to near the competitive price and (2) period prices also converge across periods. Neither property manifests in the current implementation.

We begin our analysis of market outcomes by presenting the price processes observed in the different experimental markets. Fig. 3 presents the time-series of the instantaneous prices for commodities X_1 and X_2 along with the period-average prices from the 021211 Clockwise and 030428 Counterclockwise treatment sessions.[15] The figure demonstrates clear variability of prices both for each individual transaction, as well as for period average prices, and suggests a cyclical tendency in the relative prices of the two commodities.

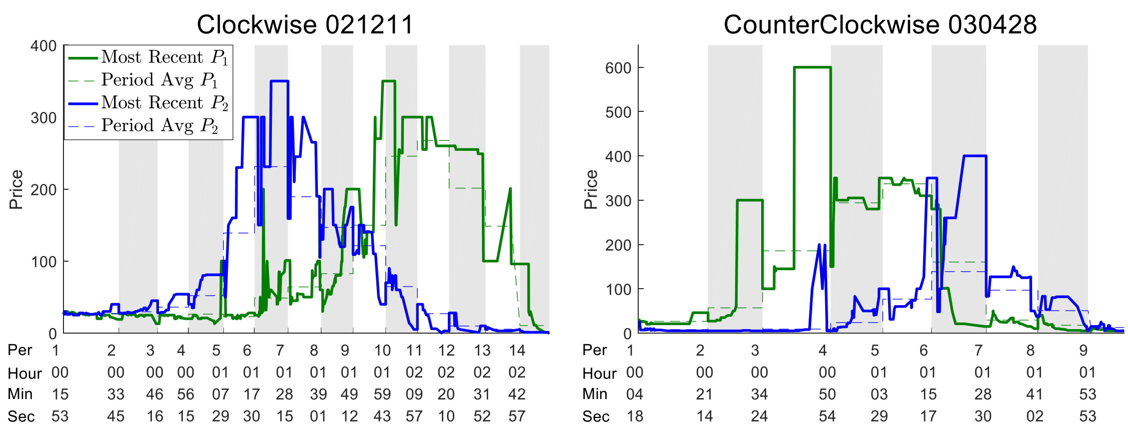

Fig. 3: Transaction Price Time Series

[14] From an assumption that a constant market price exists in the market, as if called out by a "Walrasian auctioneer", the redemption values and costs can be used to compute a market demand function and a market supply function from which a single, market clearing price can be computed. Convention has named this price the "competitive equilibrium price".

[15] These two sessions illustrate well our main experimental findings. For the sake of parsimony, the text will present several figures using these only two examples to illustrate the observed dynamics of trading and allocations. For completeness, figures from all sessions are presented in Appendices 2 & 3.

5.1. Period Price Dynamics Relative to Excess Demand Phase Diagrams

We now evaluate price dynamics in the context of the implied phase diagram constructed from excess demand in Fig. 2. Figure 4 plots average period prices in the (P_1, P_2) plane, in relation to the phase diagram and predicted dynamics of the classical model. In addition to period-average prices for the 021127 Clockwise and 030130 Counterclockwise sessions, which discretely smooth out much of the variability from individual transaction prices, Fig. 4 plots an exponentially weighted moving average of prices, a smoothed presentation characterizing the instantaneous variability in prices for individual trades. While the movement does not appear to be toward the equilibrium in either treatment, the general pattern appears consistent with classical predictions.

Figure 5 plots the period average prices for all sessions with the clockwise and counterclockwise treatments in the partitioned phase diagram. These paths present the clear impression that prices move in the general direction predicted by theory and provide a convenient illustration of our major results. First, prices in continuous double auctions need not converge to an interior equilibrium. Second, disequilibrium price movements are reasonably well-predicted by measures of excess demand.

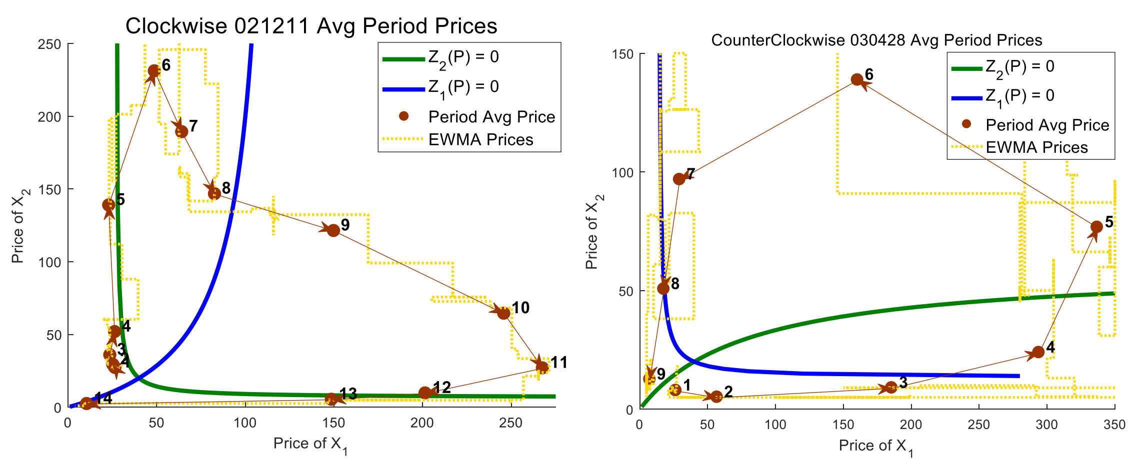

Fig. 4: Period Average Prices and Phase Diagram

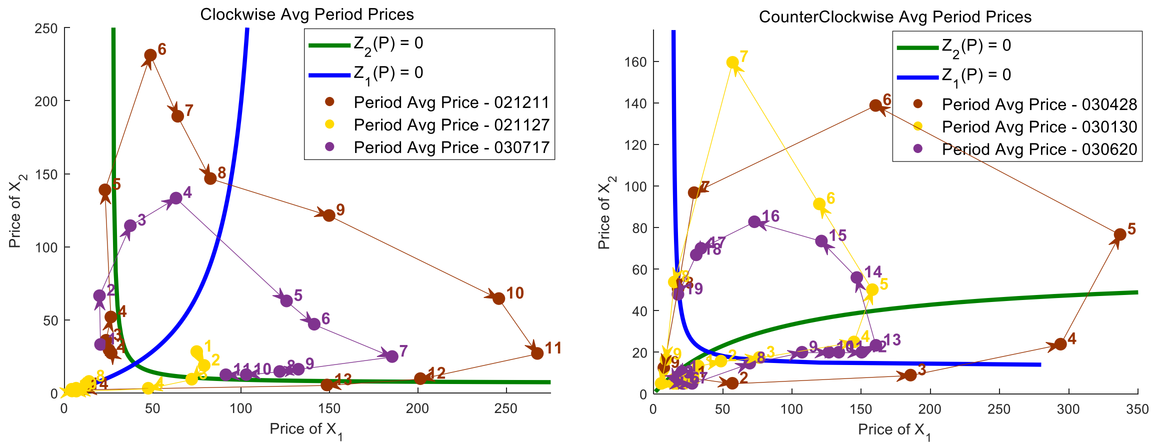

Fig. 5. Average Period Prices All Experiments

A detailed evaluation of predicted price movements by excess demand reveals some empirical limitations, as suggested by visual inspection of the Counterclockwise 030428 session period-average prices. At the beginning, price movement proceeds downward and to the right as predicted, but in period three the price of X_2 moves upward slightly, pulling prices across the partial equilibrium line for X_1 and causing a jump in phase. Continuing to follow the progression of this series, notice that the price of X_2 declines between periods 6 and 7 when the model suggests that it should continue increasing.

5.2. Price Divergence from Equilibrium

Many subtle patterns exist in these data, some of which require a generalized classical model for consistent interpretation. The empirical challenge is to separate predictable patterns in price movements from the noise inherent to market mechanisms involving real-time trading. The next sections demonstrate these results formally, statistically evaluating the degree to which equilibrium prices and excess demand predict price movements. We close this section presenting statistical evidence that disequilibrium trading persists throughout the experimental session.

Our analysis begins by considering the Euclidean distance between a pair of transaction prices and the interior theoretical equilibrium price of $P^* = (40,\ 20)$.[16] Figure 6 presents the time series of these distances for each of the six sessions, demonstrating that prices can move very far away from equilibrium in the course of trading.

[16] Because transactions occur asynchronously, we interpolate prices between trades simply using the last price at which the commodity sold.

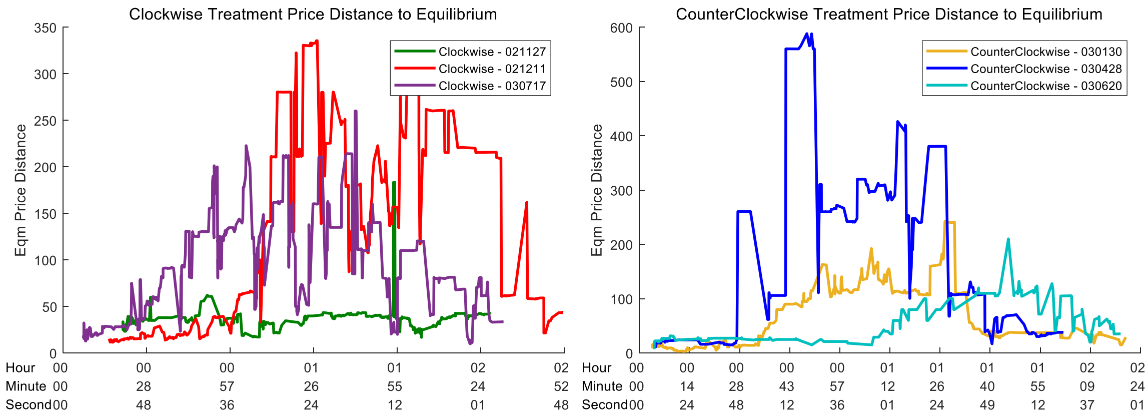

Fig. 6: Distance from equilibrium for each session and Treatment

Table 5: The Time Trend of Equilibrium Price Distance

Panel A: Pooled Regression Results

	Intercept	Trend
Coefficient	24.81	1.10
SE	8.03	0.25
p-value	<0.01	<0.01

Panel B: Individual Session Regression Results

Clockwise Treatments

021127	Intercept	Trend
Coefficient	34.39	0.03
SE	1.79	0.02
p-value	<0.01	0.13
021211	Intercept	Trend
Coefficient	13.79	1.32
SE	43.66	0.55
p-value	0.75	0.02
030717	Intercept	Trend
Coefficient	72.61	0.27
SE	25.06	0.30
p-value	<0.01	0.37

Counterclockwise Treatments

030130	Intercept	Trend
Coefficient	45.08	0.21
SE	29.20	0.34
p-value	0.12	0.54
030428	Intercept	Trend
Coefficient	121.99	0.25
SE	73.50	0.98
p-value	0.10	0.80
030620	Intercept	Trend
Coefficient	15.18	0.56
SE	8.17	0.17
p-value	0.06	<0.01

To investigate this property statistically, we estimate the time trend for theoretical equilibrium distance and test the null hypothesis that this trend is weakly negative in Table 5. Panel A presents the results from pooling all sessions of the experiment, with a significant positive trend demonstrating the tendency of transaction prices to move away from the equilibrium. Regression results from individual sessions presented in Panel B largely agree with this tendency, though the smaller sample sizes in each session prevent tests from achieving the statistical significance of the pooled sample. The weakest demonstrated trend appears in Session 021127, which was the sole session where training period prices were not fixed at (25, 25) and prices failed to move away from the origin.[17]

We summarize our conclusions from this subsection in the following result:

> **Result 1: Price Divergence from Equilibrium**
> Transaction prices do not converge to theoretical interior equilibrium but instead demonstrate a trend that moves away from equilibrium prices as time progresses and is evident over time across periods and within each period.

6. Inventories, Excess Demands, and Allocative Efficiency

In this section, we evaluate the realized allocation efficiency despite disequilibrium pricing and patterns of trading volume in these markets. While the potential issues to study here are broad ranging with a long history initiated by debates between Edgeworth and Walras, as highlighted by discussion in Walker (1987) and Donzelli (2009), our focus is narrow.

As the previous subsection demonstrated, traded prices need not converge to the competitive equilibrium as defined by parameters and can systematically move away from an interior theoretical equilibrium, a feature reflecting the views of Edgeworth. That being the case, the question turns to why the trading stopped at the end of the periods. (1) Did a different equilibrium emerge as a result of disequilibrium trades, (2) did the market largely realize allocative efficiency and further gains required complex trades (3) were market participants actively trading but the period ended because of arbitrarily imposed time limits, or (4) some other reason caused trading to end?

We first explore the dynamics or excess demand after accounting for evolving inventories and prices. By demonstrating substantial excess demand at the end of each period, we can rule out the possibility that traders converged to a different equilibrium.

We find that the market allocations' efficiency is high for complex markets, despite some gains from trade persisting at the end of each trading period throughout the experiment. The literature points to the possible exhaustion of gains from trade as an important variable. If the market operates as an efficiency seeking mechanism it would stop when

[17] To confirm that this result is not driven primarily by across-period variation in prices, we conducted a paired t-test on the beginning and end prices of the market across all sessions. The test asks if prices are closer to the equilibrium prices, (40, 20), in the first transactions executed at the beginning of the period than they are in the last transactions executed at the end of the period. For these prices, the mean distance was 18.01 (in units of X_3) at the beginning of the experiment, and 36.89 at the end ($t = 5.59$). The test is significant at $p<0.01$ for a one-tailed test. The result verifies that equilibrium divergence occurs within each trading period as well as over time across periods.

gains no longer exist. The possibility is posed by Mukherji and Guha (2011) and by Mukherji (2012) who establishes the possibility that equilibration can emerge through holdings modifying, disequilibrium, exchanges such that a competitive equilibrium exists given the holdings of the moment.

Of course, trading could have ended because the time allowed for trading ended. Recall, this is a real time market process. In the present study, this result is not driven by design decisions restricting the length of each trading period, but rather as a consequence of market participants' behavior. Analyzing the volume of trades near the end of a period demonstrates that trading activity effectively ceased well before a period ended.

6.1. End-of-Period Allocations and Efficiency

The concept of efficiency in experimental markets was introduced by Plott and Smith (1978). "Social benefits" are typically defined as sum of the redemption values of buyers from the contracts of which they are a part and the "social cost" are the cost to sellers of supplying those units. Efficiency is the actual difference between social benefits and cost in a period divided by the maximum possible given the redemption values of buyers and cost of the sellers. The efficiency measures suggest that gains from trade become exhausted, which is sometimes viewed as a form of equilibrium. In simple experimental markets, the Plott and Smith (1978) efficiency measure typically approaches 100% after several periods.

Gains from exchange in the experiment are measured in terms of additions to "take home" money acquired by trading initial endowments for other commodities. In the Scarf environment the initial endowments are worth nothing in terms of the money received in terms of the financial incentives used to induce preferences. Traders are endowed with units of commodity with no value unless complement commodities are also held. Exchange of commodities can increase the amount of money that the subject makes from the experiment. The exchanges produce income or wealth that can be summed and interpreted as net social benefits in a cost-benefit sense. At the interior competitive equilibrium, the total dollar earnings of subjects are at a maximum.

We evaluate end-of-period allocation efficiency using three devices. We evaluate allocative efficiency relative to equilibrium by totaling the dollar earnings for all agents and comparing that to the total dollar earnings realized at the competitive equilibrium allocation, which is the maximum possible. To consider Pareto efficiency, we define an agent's "residual holdings" as their inventory of a commodity providing zero marginal utility. Since these units might be valuable to someone else, they represent wasted potential. We total these residual holdings across agents and compare them to the total endowment for the market, so the residual can be expressed in percentage terms across goods. Finally, we (uniformly) randomly reallocate the residual units and calculate the average total dollar earnings available under random reallocation. Comparing the realized earnings to this average measures the allocative efficiency relative to random reallocation.

Figure 7 presents the end-of-period efficiency and residual holdings across all sessions. Averaging across all sessions, the end of period allocations realized approximately 75% of the total dollar earnings relative to the equilibrium allocation. While this level is lower than usually found in single market experiments, the level is comparable to efficiencies observed in experiments with multiple markets. The average residual holdings were approximately the same for each security, at around 25%, though Fig. 7 reveals variation in that average across periods and sessions. Randomly reallocating these residual holdings, however, does not generate substantial welfare, as the efficiency relative to random reallocation averages 94% across all sessions.

While trading exhausts much of the gains from trade, the result suggests that gains from trade exist at the end of each session that are not realized by subjects in the experiment. Trading did not stop because of equilibration due to a complete lack of gains from exchange or that gains from exchange were completely exhausted. This feature is also supported by the persistence of instantaneous excess demand and supplies at the end of periods noted in the previous subsection. Strictly speaking, the markets were not at a competitive equilibrium given their holding at the end of the period.

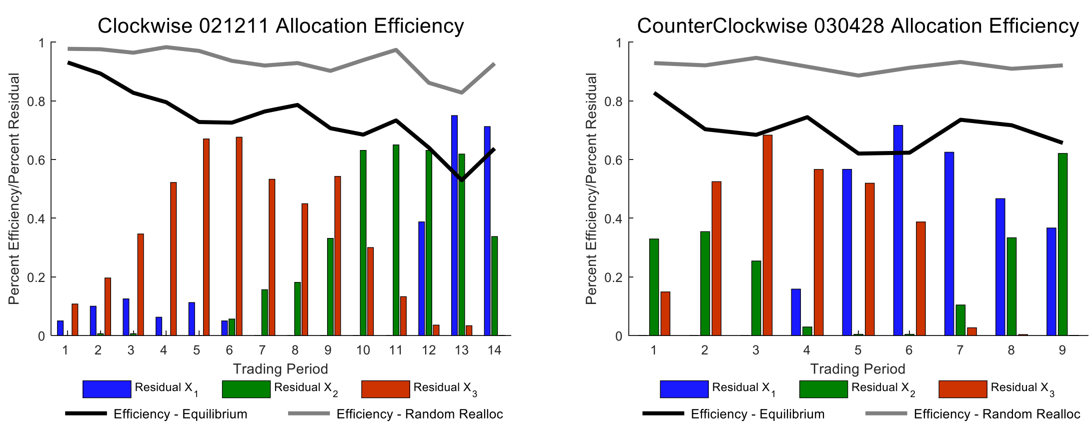

Fig. 7: Period-End Allocation Efficiency

Trading did not stop due to the arrival of the end of the period and insufficient time to trade. Figure 8 decomposes trading volume for each period into transactions that occur in the first minute, last minute, and the intervening duration of the period. Transactions occur in the last minute of a period in only 35% of the periods from the experiment and those transactions that do occur tend to be small in total value. Importantly, the failure of the markets to realize these gains from trade is not due to a design decision limiting the duration of each trading period, but rather to agents' decisions to cease transacting.

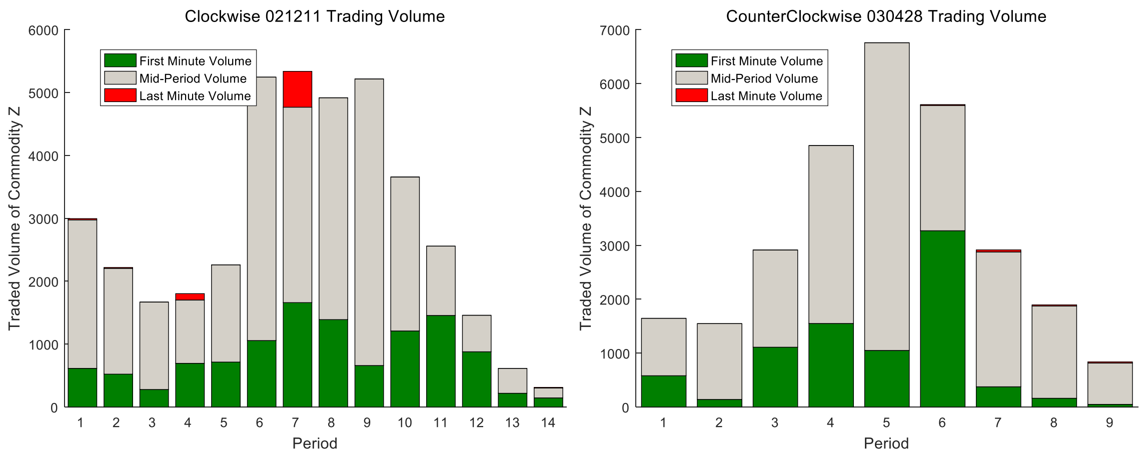

Fig. 8: Early-vs-Late Transaction Volume by Period

Result 2: Unrealized Gains from Trades Persist
Trading did not stop because gains from exchange were completely exhausted or because no time for trading remained.

A different understanding of why trading stopped is suggested by the complex transactions required to realize those gains. The required transactions are non-trivial, requiring multiple counter-parties. Each agent type realizes utility from only two of the three commodities and, at the end of each period, very few agents hold inventories of the commodities from which they receive zero utility. Across all sessions, only 14.6% (17.3%) of periods ended with an agent of type 1 (type 2) maintaining a dispreferred position in X_1 (X_2) from which they receive no utility regardless of their other holdings. Of those sessions that end with agents holding dispreferred positions, these holdings are predominantly held as shares of the numeraire commodity X_3 by the type 3 agent who receives no utility from it but is unable to purchase shares of (perhaps both) other goods at prevailing prices.

These positions limit the availability of bilateral trades that could enhance net social benefits, implying trilateral transactions would be required to realize allocative efficiency. Given this complexity and need for coordination, trading volume dissipates near the end of the period. This interpretation supports the Mukerji (2012) model of equilibria emerging from dynamic adjustments. It also suggests the possible importance of general equilibrium adjustments as excess demand order flow in complementary markets create expectations that units will become available and thus create value in units held in a given market. As suggested by Kiyotaki & Wright (1989), a universally accepted medium of exchange could facilitate arranging the trades and sequencing necessary to realize the residual gains from trade.

7. Models of Non-Equilibrium Dynamic Adjustment

The previous sections establish that much of the observed market dynamics are driven by disequilibrium trades. The extensive disequilibrium trading at a variety of prices and inventories provides a valuable setting in which to characterize price adjustment processes empirically, both using nonparametric tests and structural estimation of the price formation process.

We begin this characterization by demonstrating the classical model's predictions are consistent with observed differences in the price dynamics between the clockwise and counterclockwise treatments. Using a nonparametric test comparing these two treatments, we establish that disequilibrium price changes occur in the direction predicted by theory.

Finally, we estimate the parameters for the generalized classical model, finding limited evidence that excess demands in one market influence price dynamics in the other market. This last finding provides novel empirical evidence for the applicability of partial equilibrium modeling in a setting were general equilibrium adjustments exist.

7.1. Excess Demand Dynamics Predict Price Movement Direction

Our analysis in the previous sections suggests that prices from the double auction mechanism diverge from theoretical equilibrium prices. We now present evidence that models based on excess demand accurately predict the direction of price movements, as suggested by the phase diagrams in Figs. 4 and 5. We consider here two nonparametric tests to verify the link: a "clockhand" test that applies to individual transaction data and a "sign" test analyzing period-level price dynamics. These tests exploit variation between the Clockwise and Counterclockwise treatments to demonstrate one sense in which price movement is away from the competitive equilibrium.

The "clockhand" test simply recenters prices so that equilibrium lies at the origin and measures the angle in price space between where the data started relative to the equilibrium and where the prices are at any instant of time. A line segment connecting current prices to the equilibrium functions as the hand of a clock, and as prices change, that line segment rotates around the equilibrium. Anderson, et. al., (2004) presents a geometric interpretation of this analysis wherein the clockhand test measures the accumulated rotation of prices over time. As a non-parametric test robust to both boundary restrictions and asynchronous trades, the clockhand test can incorporate the entire time-series of individual transactions from all periods and sessions.

Figure 9 shows the cumulative angle changes based on individual transactions in all 6 sessions. There is a clear separation between the clockwise and counterclockwise treatments. In addition, note that 2 of the counterclockwise treatments resulted in cumulative angle changes greater than 2π. i.e., in two of the sessions, the price orbit completed one cycle.

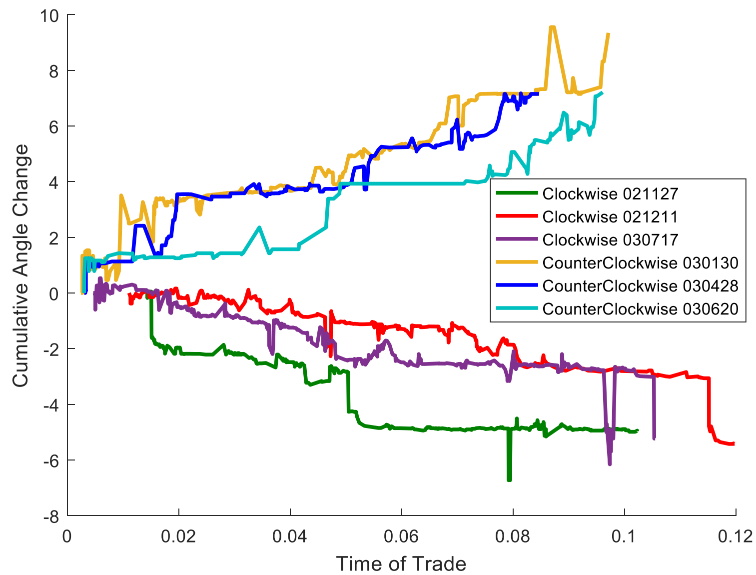

Fig. 9: Clockhand Model Plotting Cumulative Angle of Price Changes

The sign test is a simple binomial test that counts the instances where the sign of the price change in a given trade matches the sign of the excess demand in both markets, i.e., whether $\mathrm{sign}(P_{1,t} - P_{1,t-1}, P_{2,t} - P_{2,t-1}) = \mathrm{sign}(Z_1(P_{t-1}), Z_2(P_{t-1}))$. Under the null hypothesis that price dynamics are not predictable by excess demand, this event has a 0.25 probability of occurring. This test is sensitive to both boundary effects *and* asynchronous trades and, therefore applies to individual transactions as well as period average prices. Pooling over all sessions, there were a total of 124 data points, of which in 52 trials the price change was predicted correctly by excess demand in both the X_1 and X_2 markets. Compared to a random prediction expecting 31 correct predictions, the test was significant at a p = 4.11e-5.

Result 3: Excess Demand Dynamics Predict Price Movements
Prices tend to move in the direction predicted by excess demand, both at the individual transaction level and across periods over time.

7.2. Accommodation and Linkage in the Absolute Model

We now evaluate the predictive power of the Generalized Absolute Model in characterizing price dynamics. Having demonstrated that excess demands predict the direction of price movement, we now test whether they also effectively predict the magnitude of observed price changes and evaluate the significance of market linkages. The Generalized Absolute Model applies to the experimental data taking Sect. 3. C's difference Eq. (2) with the instantaneous excess demand function as the conditional expectation for transactional price changes in structural estimation equations:

$$\begin{pmatrix} P_{1,t} - P_{1,t-1} \\ P_{2,t} - P_{2,t-1} \end{pmatrix} = \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{pmatrix} Z_{1,t-1}(P_{t-1}) \\ Z_{2,t-1}(P_{t-1}) \end{pmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}. \tag{4}$$

The noise term $\varepsilon_t \equiv \left[\varepsilon_{1,t}, \varepsilon_{2,t}\right]'$ is assumed to be mean zero and satisfies the usual conditions for consistent regression analysis.

We present results from estimating the econometric models equation-by-equation allowing intercepts to vary across sessions, using Feasible GLS to account for residual autocorrelation and Autoregressive Conditional Heteroscedasticity within each session. The regressand corresponds to price changes at the transaction level, winsorized absolute price changes for X_1 and X_2 at 50 and 25 units of X_3, respectively, to control the influence of outliers.[18] The regressors consist of excess demand measured instantaneously based on the last available market prices and inventories.

Table 6: Estimated Absolute Model Excess Demand Coefficients and Significance

	Coefficient	SE	t-Statistic	p-Value
a_{11}	5.41 E-02	1.63 E-02	3.32	<0.01
a_{12}	1.32 E-02	8.97 E-03	1.47	0.14
a_{21}	6.07 E-03	1.00 E-02	0.60	0.55
a_{22}	1.47 E-02	4.71 E-03	3.11	<0.01

Table 6 presents the estimated coefficients and associated significance measures for the Instantaneous Excess Demand of X_1 and X_2, with three key findings regarding the

[18] The results presented here model residuals according to an AR(3) process with Autoregressive Conditional Heteroskedasticity based on standard specification tests analyzing the partial autocorrelation function of the residuals. We evaluated several other specifications, including OLS, Seemingly Unrelated Regressions, and fixed effects, as well as observational weighting (quantity weighted and time-weighted), with qualitatively similar results that differ mainly in coefficients' estimated standard errors. Introducing session-level fixed effects for coefficients results in noisy estimates as the specification fails to take advantage of the information available from the different excess demand dynamics in the clockwise and counterclockwise treatments. For all tables using pooled results, we present session-level results in Appendix 3.

classical model. First, Instantaneous Excess Demand for both goods are significant drivers of own-price changes, i.e., the coefficients a_{11} and a_{22} are both statistically significant and positive. Second, neither of the off-diagonal coefficients, a_{12} and a_{21}, are significant, suggesting excess demand conditions in one market has a negligible effect on price dynamics in the other. Calculating an F-Test for the joint restriction, $a_{12} = 0 = a_{21}$ in the SUR specification is only weakly rejected at the 10% level with a p-Value of 0.08.

We can use the estimates in Table 6 to evaluate whether Walras' Fundamental Principle, which states that the expected sign of a commodity's price change should match the sign of its excess demand, holds in expectation. At every observed transaction, we compute the sign of the commodities' predicted price changes from the Generalized Absolute Model and compare those with the sign of their Instantaneous Excess Demands. If Walras' Fundamental Principle holds in expectation according to the Generalized Absolute Model, these signs should match and we observe this consistency in 69% of the sample observations.

Combined, these observations suggest supporting evidence for the Classical Restrictions in the Generalized Classical Model. In particular price changes reflect own excess demand and not the excess demand in other markets as postulated by partial equilibrium theories. The next result summarizes the findings of this subsection.

Result 4: Absolute Model Estimates Support the Classical Model
The estimated coefficients in the Generalized Absolute Model are consistent with the Classical Model's restrictions:

- Excess demand for a good has a significant impact on expected price changes for that good, supporting price adjustment models driven by partial equilibrium dynamics.

- Cross-excess demand coefficients in the adjustment matrix are much smaller than own-excess demand coefficients and the hypothesis restricting these (off diagonal) coefficients to be zero is not rejected at conventional significance levels.

- Walras' Fundamental Principle that the expected sign of a commodity's price change aligns with the sign of its excess demand is violated only in states of extreme disequilibrium and satisfied in 69% of the sample observations.

7.3. Accommodation and Linkage in the Relative Model

We now present Relative Model's structural equations in a regression form, scaling excess demand for each good by the initial endowment allocated to each type of market participant:[19]

$$\begin{aligned} \frac{P_{1,t}-P_{1,t-1}}{P_{1,t-1}} &= a_{10}+a_{11}\frac{Z_{1,t-1}\left(P_{t-1}\right)}{20}+a_{12}\frac{Z_{2,t-1}\left(P_{t-1}\right)}{40}+\varepsilon_{1,t} \\ \frac{P_{2,t}-P_{2,t-1}}{P_{2,t-1}} &= a_{20}+a_{21}\frac{Z_{1,t-1}\left(P_{t-1}\right)}{20}+a_{22}\frac{Z_{2,t-1}\left(P_{t-1}\right)}{40}+\varepsilon_{2,t} \end{aligned} \tag{5}$$

From the structural regression Eq. (5), we can apply the same estimation strategy adopted in the previous section to the Relative Model. The estimated parameters appearing in Table 7 demonstrate a similar relationship to the Relative Model's results in Table 6. The adjustment matrix coefficients for own excess demand (a_{11} and a_{22}) are much larger than those on cross-excess demand (a_{12} and a_{21}), presenting material support for partial equilibrium adjustment dynamics.

Table 7: Estimated Relative Model Excess Demand Coefficients and Significance

	Coefficient	SE	t-Statistic	p-Value
a_{11}	2.56 E-02	4.73 E-04	5.41	<0.01
a_{12}	1.44 E-02	5.20 E-04	2.78	<0.01
a_{21}	0.85 E-02	5.42 E-04	1.56	0.12
a_{22}	2.46 E-02	5.10 E-04	4.82	<0.01

In this specification, these cross-excess demand coefficients are estimated with sufficient precision to statistically reject the dominant diagonal restriction. The off diagonal elements are not zero. While the partial equilibrium model receives support, the linkages of market excess demands (the general equilibrium adjustments) can be detected in this specification. As an empirical phenomenon, these adjustments could arise from phenomena that aren't included in the abstract model, notably in how expectations of future liquidity could be informed by excess demand in other markets.

The estimated coefficients suggest that partial equilibrium influences will dominate the excess demand linkage influences so long as disequilibrium does not generate severe imbalances in excess demands. Using the estimated expectations from the Generalized Relative Model, we evaluate whether Walras' Fundamental Principle holds following a similar analysis from the previous subsection. For each transaction, we compute the sign of a commodity's predicted price change from the Generalized Relative Model and find

[19] This treatment allows both the regressand and regressor to have similar scale. We estimate the model using FGLS with an AR(3) model and ARCH for the residuals, winsorizing relative price changes at 50% to mitigate the influence of extreme outliers.

this empirical estimate of the expected price change matches the sign of the commodity's Instantaneous Excess Demand in 81% of the transactions.

Result 5: Relative Model Estimates Statistically Reject Classical Restrictions
The estimated coefficients in the Generalized Relative Model statistically deviate from Classical Model restrictions while supporting Walras' Fundamental Principle:

- Excess demand for a good has a significant impact on expected price changes for that good, supporting price adjustment models driven by partial equilibrium dynamics.

- Cross-excess demand coefficients in the adjustment matrix (excess demand linkages) are much smaller than own-excess demand coefficients though the hypothesis restricting these coefficients to be zero is rejected at conventional significance levels.

- Walras' Fundamental Principle that the expected sign of a commodity's price change aligns with the sign of its excess demand is violated only in states of extreme disequilibrium and satisfied in 81% of the sample observations.

8. Conclusion: Interpretations and Implications

In closing, we observe that we avoided criticisms grounded on the possible lack of empirical content of general equilibrium theory. The feasibility of evaluating the empirical properties is achieved through a predictive analysis based on the information sets available when studying markets in controlled environments as opposed to those found naturally occurring. By directly observing both price changes and the traditionally unobservable aggregate excess demand, we can directly evaluate whether the predictions from models of price formation in general equilibrium match market outcomes. Our results affirm the predictive power of these models.

This paper explores multi-market price dynamics under challenging conditions of an unstable equilibrium in which prices do not converge to the unique interior equilibrium predicted by traditional equilibrium models. However, while equilibrium is not observed, the traditional models provide predictions of price dynamics driven by the excess demand functions. The predictions allow the experimental study of empirical properties of underlying principles of dynamics. We discover non-convergence in experimental markets, with transaction prices moving away from an interior equilibrium as predicted by excess demands and with gains from trade persisting throughout the experiment session. Thus, we find that frictionless models originally motivated by price equilibration provide a useful predictive model of non-equilibrium price dynamics. These results underscore the positive value of equilibration dynamics for economic analysis in multiple market settings even in settings that do not satisfy all assumptions underlying equilibration.

In estimating models of price dynamics, we are able to test the sensitivity of prices to disequilibrium in outside markets. In so doing, we are able to quantify the importance of partial equilibrium adjustments on prices from shocks to excess demand for that good relative to general equilibrium linkage adjustments on prices from shocks to excess demand for other goods. Though statistical estimates reject the absence of general equilibrium linkage adjustments, their estimated magnitudes are small compared to the first-order partial equilibrium adjustments. As a test of Walras' Fundamental Principle, we find the theoretically expected sign of a commodity's price change accords with the sign of its excess demand in 81% of the sample.

These results have implications that extend beyond testing theories of disequilibrium price dynamics. In contrast to previously reported patterns, we demonstrate that (1) prices in the commonly adopted continuous double auction with multiple markets need not converge, (2) need not exhibit movement toward an equilibrium and (3) may not realize efficient allocations on the order of magnitudes often found in experiments with single markets. The results lead to insights about the principles that govern disequilibrium dynamics coordination and the role of multiple markets in equilibration. Our findings also suggest that the excess demand dynamic theories of equilibration can be useful in identifying the conditions under which phenomena (1) to (3) may occur.

In applied settings ranging from financial markets to industrial organization, economic analysis often explicitly or implicitly relies on ignoring general equilibrium adjustments on isolated markets. It is difficult to conceive how researchers could practically account for the interdependent phenomena in the field. However, experimental markets provide a viable setting for exploring the relative magnitude of partial and general equilibrium adjustments in economic analysis. For now, the experiments provide data verifying Walras' Fundamental Principle under the demanding conditions and invite exploration to other settings.

9. APPENDIX 1: Theoretical Background and Experimental Details

9.1. Notes on Experimental Design and Parameters

The parameters chosen for the experiments reflected considerable research on the various possibilities. This appendix provides an overview that attempts to help the reader understand the parameters used and provides those interested with suggestions about additional experiments and tests.

Four parameters are used to form preferences and initial endowments across the experimental series. These parameters $\{\alpha, \beta, \gamma, q\}$ interacted with preferences and endowments. The interactions with preferences are as follows. Notice that α is a scaling parameter for X_2, β is a scaling parameter for X_3 and γ is a scaling parameter for X_1. The parameter q operates on individuals to change the value of different goods across the individuals. The functions studied when in parametric form are:

$$U_1 (X_2, X_3) = \min [X_2/q\alpha, X_3/\beta]$$
$$U_2 (X_1, X_3) = \min [X_1/\gamma, X_3/q\beta]$$
$$U_3 (X_1, X_2) = \min [X_1/q\gamma, X_2/\alpha]$$

The choice of experimental design also involves an interaction of the four parameters with initial endowments. The following example illustrates the material that will be presented in the table in the next section. The example is for the case of clockwise unstable parameters that were actually used in the experiments.

Clockwise Parameters: $(\gamma, \alpha, \beta, q) = (20, 40, 800, 1/3)$

	Type 1	Type 2	Type 3
Preferences:	$\min\{3X_2/40, X_3/800\}$	$\min\{X_1/20, 3X_3/800\}$	$\min\{3X_1/20, X_2/40\}$
Endowments:	$E_1=(0,0,\beta)=(0,0,800)$	$E_2=(\gamma,0,0)=(20,0,0)$	$E_3=(0,\alpha,0)=(0,40,0)$

The predictions for this set of parameters are:

1. Equilibrium: $(P_1, P_2) = (\beta/\gamma, \beta/\alpha) = (40, 20)$
2. Dynamics: Unstable time path moving in a clockwise direction

Table 8 provides a pattern of parameters that created a background for the specific choice of parameters for implementation. Parameters that theoretically lead to closed cycles and to stable paths have been studied by Anderson, et. al., (2003) and by Plott (2001). While existing studies did not use the parameters in the table, the parameters used in those studies did lead to the same qualitative implications for system behavior as the parameters in the table. Thus, we make no attempt here to study parameters that theoretically lead to stability or theoretically lead to closed cycles. The question posed here is whether or not divergence can be observed in practice so the focus was on parameters that theoretically lead to divergence. Those parameters correspond to specifications in the upper left and lower right of Table 8, below. As can be seen, the difference between the clockwise and counter-clockwise treatments resides in the choice of *q* and the choice of initial endowments.

Table 8: General Parameter Set for Stability Analysis

Q	Endowments $(\gamma, \alpha, \beta) = (20,40,800)$	
	type one $(\mathbf{x_1,x_2,x_3}) = (0, \alpha, 0)$ type two $(\mathbf{x_1,x_2,x_3}) = (0, 0, \beta)$ type three $(\mathbf{x_1,x_2,x_3}) = (\gamma, 0, 0)$	type one $(\mathbf{x_1,x_2,x_3}) = (0, 0,\beta)$ type two $(\mathbf{x_1,x_2,x_3}) = (\gamma, 0, 0)$ type three $(\mathbf{x_1,x_2,x_3}) = (0,\alpha,0)$
q>1 q = 3 for experiments	unstable counterclockwise equilibrium prices (40,20)	Stable equilibrium prices (40,20)
q=1	limit cycle counterclockwise equilibrium prices (40,20)	limit cycle clockwise equilibrium prices (40,20)
q<1 q = 1/3 for experiments	Stable equilibrium prices (40,20)	unstable clockwise equilibrium prices (40,20)

Table 9 contains the parameter set for the experiments conducted. The information in this table is essentially the same as the information in Table 1 in the text. It is included here for the convenience of readers who want to compare the parameters that were implemented to the more general possibilities.

Table 9: Preferences and Endowments

	Type i = 1,2,3	$U^i(x_i,y_i,z_i)$	endowments $\omega_1 = (\mathbf{x_i,y_i,z_i})$	Remarks
Clockwise: q=1/3, $(\gamma,\alpha, \beta) = (20,40,800)$; Equilibrium $P_x = \beta/\gamma$, $Py = \beta/\alpha$				
	1	min{3y/40,z/800}	ω_1=(0,0,800)	The classical model predicts divergence with tendencies in a clockwise direction.
	2	min{x/20,3z/800}	ω_2=(20,0,0)	
	3	min{3x/20,y/40}	ω_3=(0,40,0)	
Counterclockwise: q = 3, $(\gamma,\alpha , \beta) = (20,40,800)$				
	1	min{y/120,z/800}	ω_1=(0,40,0)	The classical model predicts divergence with tendencies in a counterclockwise direction.
	2	min{x/20,z/2400}	ω_2=(0,0,800)	
	3	min{x/60,y/40}	ω_3=(20,0,0)	

9.2. Initial Conditions and Cyclical versus Explosive Behavior

The cyclical price patterns depicted in Fig. 2 depend in part on the initial conditions, which in the current market implementation could instead give rise to explosive price patterns. Excess demand dynamics in a continuous market without noise presented in Fig. 10 demonstrates the potential for explosive, rather than cyclical price dynamics depending on where prices initiate. Notably, from the training price conditions (25, 25), the clockwise model predicts such an explosive dynamic. Further, transactions occur at prices throughout the price space over the course of the entire experiment, a result that's incompatible with the precise predictions of the difference equations.

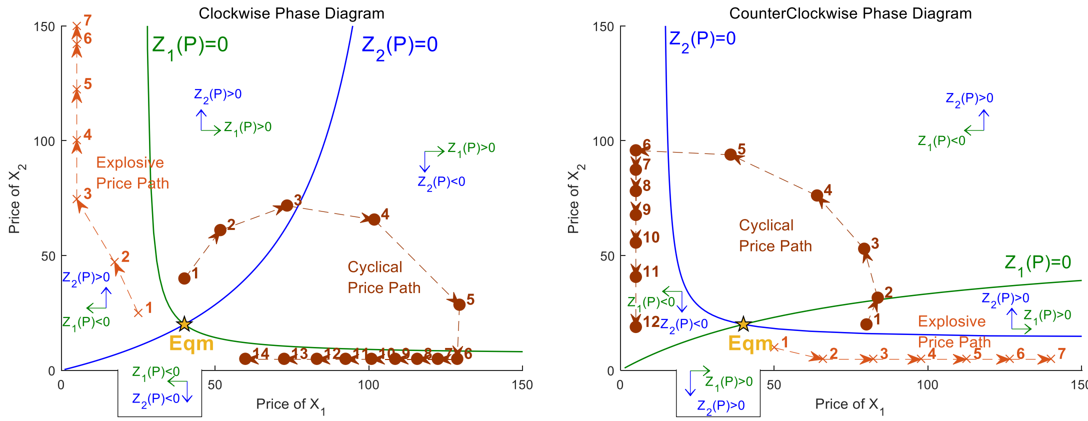

Fig. 10: Cyclical and Explosive Price Paths

In practice, three features of the markets implemented in the experiment can give rise to phase changes that induce cyclical price dynamics even when prices may lie in the explosive region. First, the limited number of units of X_3 in the economy function as a price ceiling for P_1 and P_2 that puts a ceiling on the degree to which explosive prices can be observed. Second, unmodeled variation in the prices at which trades execute is of sufficient magnitude to "jump phase" and move prices into a region of the phase diagram in which cyclical dynamics dominate. Third, constraining trades to integer units of X_1 and X_2 complicates the excess demand dynamics allowing for cyclical behavior to be observed from a larger set of starting conditions and substantially expanding the set of equilibrium prices.

9.2.1 Effective Price Ceilings Bound Explosive Tendencies

A simple practical feature of the markets we study prevent us from observing unboundedly high prices. In an economy with no more than 800 units of X_3, the price of any good cannot exceed 800. The highest transaction price in the observed sample occurred for $P_2 = 600$ in the explosive region of the counterclockwise treatment, suggestive of an explosive tendency in prices. Regardless, though, this explosiveness is limited by the available quantity of currency in the economy.

9.2.2 Unmodeled Variation in Price Processes

While excess demand-driven dynamics provide a reasonable model for expected price changes, it is an incomplete model and actual price changes are influenced by many other factors that are not included in the model. The residual price variation is apparent in the time series presented in Fig. 3 and the smoothed out exponentially weighted moving average prices presented in Fig. 4. Figure 11 plots unsmoothed transaction prices in the price space, demonstrating substantial variation above and beyond that which is predicted by excess-demand dynamics.

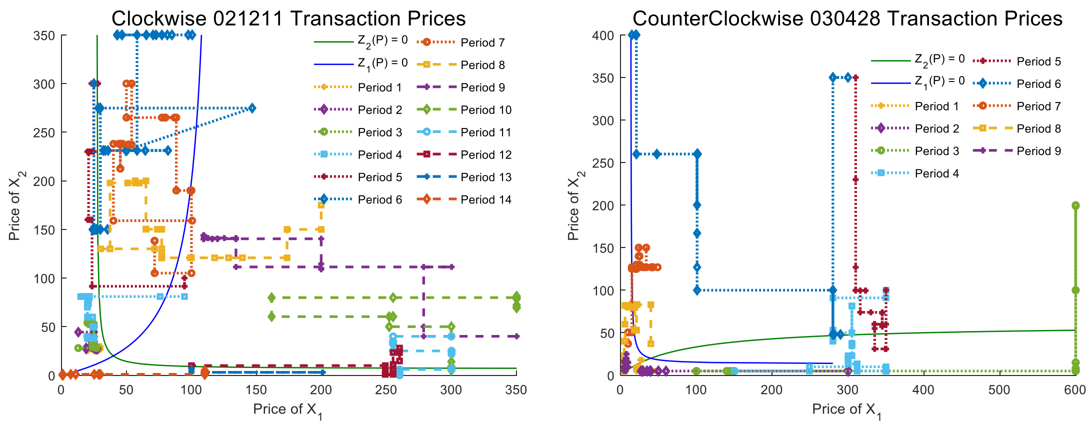

Fig. 11: Transaction Price Variability

Importantly, this unmodeled variability in prices is sufficient to systematically transition from explosive regions of the price space into the cyclical regions. To establish this, we consider calibrated simulations of the price process based on the estimated parameters from Table 12's Aggregated Demand Model, starting from several initial positions in the explosive region while varying the amount of "noise" in the process from 25% to 100% of the estimated variance. Running 10,000 simulations for both treatments, we calculate the frequency with which the price process reaches the upper bound and the frequency with which it moves into the "Cyclical Region" as characterized by price pairs lying to the northeast of equilibrium (i.e., P_1>40 and P_2>20).

Table 10 presents the results of this analysis, with essentially all simulations passing through the cyclical region and rarely reaching the price ceiling. Notably, essentially all simulations entered the Cyclical Region of price space and very rarely did the simulated price paths reach the maximum price ceilings. Across all specifications for the clockwise treatment, only 3 out of 90,000 simulations reached the price ceiling and only 5 failed to pass through the cyclical region. The counterclockwise treatment simulations had more explosive tendencies, but in the noisiest conditions less than 11% of simulations reached the price ceiling while across all specifications, over 99% of simulated price paths entered the cyclical region.

Table 10: Properties of Simulated Price Paths

Panel A: Clockwise Treatment

	Sim Variance as % of Fitted Var	Frequency of Reaching Price Ceiling 0%	25%	100%	Frequency of Entering Cyclical Regions 0%	25%	100%
Initial Prices	(50, 10)	0%	0%	0%	0%	100%	100%
	(10, 10)	0%	0%	0%	0%	100%	100%
	(25, 25)	0%	0%	0%	0%	100%	100%

Panel B: Counterclockwise Treatment

	Sim Variance as % of Fitted Var	Frequency of Reaching Price Ceiling 0%	25%	100%	Frequency of Entering Cyclical Regions 0%	25%	100%
Initial Prices	(50, 10)	0%	0%	11%	0%	100%	100%
	(10, 10)	0%	0%	9%	0%	100%	100%
	(25, 25)	0%	0%	10%	0%	100%	100%

The price dynamics in these simulations are affected by partial equilibration forces arising from a good's own excess demand as well as general equilibration forces driven by other goods' excess demands and error correction dynamics. The influence of these additional forces on expected price dynamics is demonstrated by the simulations in which the unmodeled variance of the price process is set to zero. Notably, the price paths in this simulation never reach the price ceiling, demonstrating that general equilibration forces and error correction dynamics suffice to prevent explosive price paths. Further, these additional forces are not sufficient to drive prices into the cyclical region of price space, which requires some noise in the price process to transition phases.

9.2.3 Indivisibility, Excess Demand, and Multiple Equilibria

The unmodeled variation in price dynamics need not be entirely behavioral in its origin and could arise from approximation errors in applying a theory of equilibrium based on abstract principles to a setting that doesn't strictly satisfy all the assumptions of that theory. As an example of one such approximation error, consider the simple restriction that units of all commodities are indivisible even though the theory of price adjustment assumes individuals' consumption decisions take place on a real-valued continuum of quantity and price. Theoretically accounting for this indivisibility substantially expands the set of equilibria, as the friction associated therewith. A full analysis of these considerations presents a theoretical exercise well beyond the scope of the current paper. Our intent here is simply to demonstrate the possibility for indivisibility to generate a variety of equilibria and potential price processes.

First, we explain how to define the indivisible market excess demand as well as indivisible demand under the assumption that the commodity and price spaces are constrained to be integer-valued. Then, we demonstrate how these demand functions could influence price dynamics and market equilibria, restricting attention to the case with clockwise parameters.

9.2.3.1 Defining Integer-Valued Demand

An agent's indivisible demand function is obtained by maximizing their utility subject to a budget constraint with integer-valued variables. Let us consider the first agent-type with endowments:

$$\begin{aligned} &\max U^{(1)}(X) = \min\left[3X_2, X_3/20\right] \\ &\quad \text{subject to } P_1X_1 + P_2X_2 + X_3 = M_{10} = 800 \\ &\qquad X_i \in \mathbb{Z}_+,\ i \in \{1,2,3\} \end{aligned} \tag{6}$$

For a given integer prices (P_1, P_2), let $\hat{X}_2^{(1)}$ be the utility maximizing quantity of the second in the case of ordinary real commodity space. Ignoring the indivisibility constraint, of course this quantity will generally not be integer-valued but rather a real number. Define $\underline{X}_2 = \lfloor \hat{X}_2 \rfloor$ to denote the largest integer less than or equal to $\hat{X}_2$ and also $\overline{X}_2 = \lceil \hat{X}_2 \rceil$ denote the smallest integer larger than or equal to $\hat{X}_2$. The third good quantities demanded corresponding to $\underline{X}_2$ and $\overline{X}_2$ are respectively determined by the budget equation with $\underline{X}_3 = 800 - P_2\underline{X}_2$ and $\overline{X}_3 = 800 - P_2\overline{X}_2$, each of which will be integer-valued.

This agent is supposed to choose, as integral demand, whichever bundle of goods gives the largest utility, say, $\left(\overline{X}_2^{(1)}, \overline{X}_2^{(1)}\right)$. As demonstrated by the indifference curves in Fig. 12, the utility from the consumption bundle $\left(0, \overline{X}_2^{(1)}, \overline{X}_3^{(1)}\right)$ is clearly larger than that derived from $\left(0, \underline{X}_2^{(1)}, \underline{X}_3^{(1)}\right)$, so the integer-valued demand vector is given by:

$$\operatorname{int} D^{(1)}(P_1, P_2) = \left[0, \overline{X}_2^{(1)}, \overline{X}_3^{(1)}\right] \tag{7}$$

A similar analysis applies to maximizing welfare and computing demand for the second agent-type that derives utility from a complementary combination of commodities X_1 and X_3.

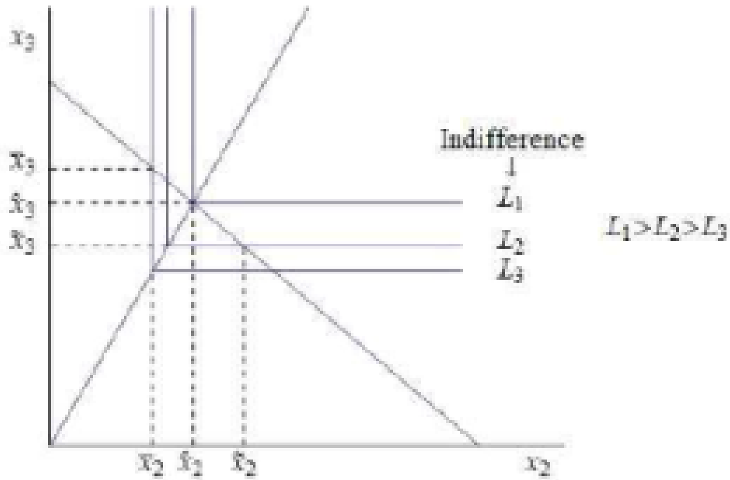

Fig. 12 Indifference Curves for Integer-Constrained Consumption Bundles

The third agent-type, which derives no utility from the numeraire good, presents a slightly more complicated optimization problem.

$$\begin{aligned} &\max U^3(x) = \min[3X_1, X_2/2] \\ &\quad \text{subject to } P_1X_1 + P_2X_2 + X_3 = M_3 = 40P_2 \\ &\qquad\qquad X_i \in \mathbb{Z}_+,\ i \in \{1,2,3\} \end{aligned} \tag{8}$$

In the divisible setting of a real commodity space, the solution to this problem is uniquely determined with $X_3 = 0$. However, when restricted to integer commodity space, the solution becomes tedious and complicated. For instance, although positive holdings of X_3 is irrelevant to his utility, the integer constraint will bring about positive holding of X_3 as a result of utility maximization and could give rise to a multiplicity of solutions. How many solutions may depend on the value of relative prices for P_1 and P_2. The following picture well illustrates these phenomena.

In Fig. 13, let $\left(\hat{X}_1, \hat{X}_2\right)$ be the optimal consumption bundle of perfectly divisible goods and define $\underline{X}_1 = \left\lfloor \hat{X}_1 \right\rfloor$ and $\overline{X}_1 = \left\lceil \hat{X}_1 \right\rceil$ as nearest integers below and above $\hat{X}_1$, respectively as in our analysis of the first agent-type's consumption problem. Further, let $\underline{X}_2 = \left\lfloor 40 - P_1\underline{X}_1 / P_2 \right\rfloor$ and $\overline{X}_2 = \left\lceil 40 - P_2\overline{X}_1 / P_2 \right\rceil$ as the smallest and largest integers of X_2 satisfying the budget constraint. This agent's demand for good X_3 is either the residual $\underline{X}_3 = P_2\left(40 - \underline{X}_2\right) - P_1\underline{X}_1$ or $\overline{X}_3 = P_2\left(40 - \overline{X}_2\right) - P_1\overline{X}_1$. Examining the figure further illustrates the multiplicity of solutions, as the bundle $\left(\underline{X}_1, X_2^*\right)$ gives the same utility as $\left(\underline{X}_1, \underline{X}_2\right)$. Therefore, demand in this case is not actually a function, but rather the correspondence that includes, in this example:

$$\text{int}\, D^{(3)}\left(P_1, P_2\right) \subset \left\{\left[\underline{X}_1, \underline{X}_2, \underline{X}_3\right], \left[\underline{X}_1, X_2^*, X_3^*\right]\right\} \tag{9}$$

To get a sense of the multiplicity of such solutions, consider that for a candidate set of prices $P = (5, 400)$ there are over 70 solutions to the third agent-type's optimization problem.

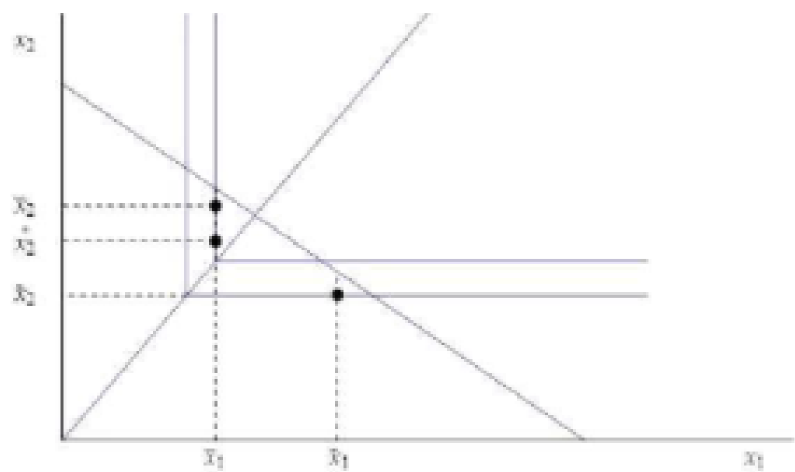

Fig. 13 Indifference Curves for Type-3's Integer-Constrained Consumption Bundles

Summing the demand functions for the first two agent-types with each of the candidate solutions to the third agent-type's optimization problem defines an excess demand correspondence:

$$\text{int}\, Z\left(P_1, P_2\right) = \sum_{i=1}^{3} \text{int}\, D^{(i)}\left(P_1, P_2\right) - \left[20,\ 40,\ 800\right] \tag{10}$$

9.2.3.2 Equilibrium and Price Dynamics with Indivisibility

We apply brute-force numerical calculation to investigate the equilibrium and price dynamics subject to integer constraints on prices. Using Mathematica for these purposes to demonstrate our results, we consider a fixed integer price set $S = \left\{(i, j) \mid i = 1,..,200; j = 1,...,500\right\}$. Given the demand correspondences from the previous subsection, the equilibrium will depend on which bundle is selected from that correspondence, requiring an additional assumption to complete the model. Here, we consider one random and two deterministic approaches to resolving this indeterminacy.

First, suppose that each agent randomly draws a single consumption bundle from the correspondence that maximizes their utility for each possible price $P \in S$. Under this specification, the set of market excess demand functions becomes vast. Denoting these market excess demand functions by random-int$Z(P_1,P_2)$, not that these provide a mapping from integer space into integer space. Consequently, Walrasian dynamics can be represented by the difference equation:

$$P_t - P_{t-1} = \text{random-int}\, Z\left(p_{t-1}\right) \tag{11}$$

Given the random selection of consumption bundles, this adjustment process will be necessarily stochastic.

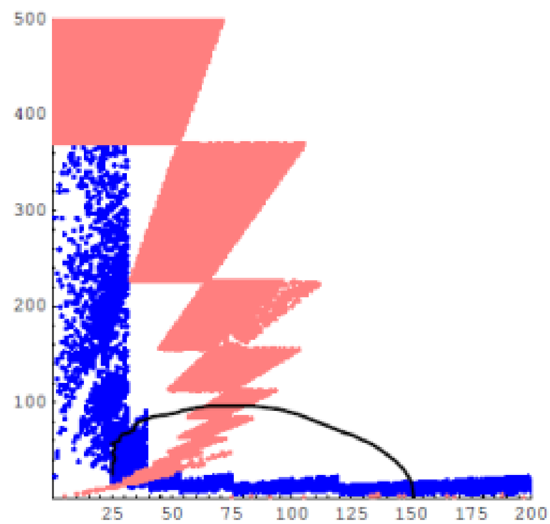

Fig. 14 Integer-Valued Partial Equilibrium and Excess Demand Dynamics with Randomly Selected Bundles from the Demand Correspondence

Figure 14 presents the set of prices for which partial equilibrium obtains in blue pixels (for which $\text{random-int}\, Z_1\left(p\right)=0$) and red pixels (for which $\text{random-int}\, Z_2\left(p\right)=0$). Consistent with the usual definition, equilibrium prices are defined by those points at which the excess demand for both goods equals zero. Under this specification, there are 228 price combinations that are compatible with zero excess demand, demonstrating the severe multiplicity of potential equilibria. The black line traces out the price path based on the price path defined by Eq. (11), which notably follows the cyclical pattern despite starting in the explosive region of the clockwise treatment. Driven by the random selection consumption bundles, the integer-constrained dynamic allows for many other potential paths.

To remove the randomness in resolving the consumption choice, consider a setting wherein agents choose the bundle from their demand correspondence with either the smallest or largest quantities of X_1. These Minimal and Maximal X_1 specifications are presented in Fig. 15 Panels A and B, respectively. The Minimal X_1 specification in Panel A contains a large number of 964 equilibrium price combinations along with a candidate price path that travels away from equilibrium before being absorbed by the boundary. The Maximal X_1 specification in Panel B includes only 78 equilibria, and suggest a price path starting from (25, 25) that will start to orbit clockwisely before approaching the boundary price for P_2.

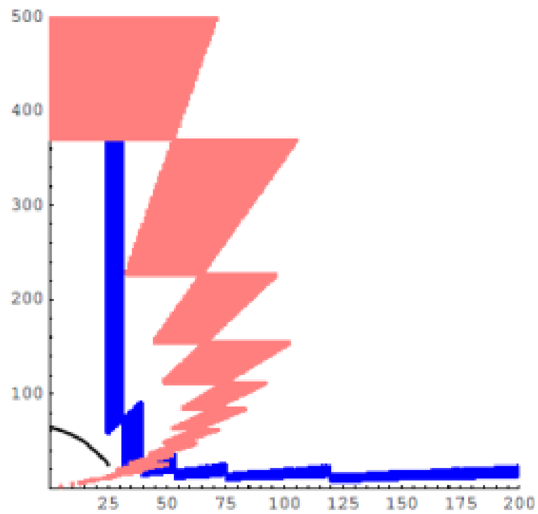

Fig. 15 Integer-Valued Partial Equilibrium and Excess Demand Dynamics with Selecting Bundles from the Demand Correspondence that Minimize X_1

The analysis here is in no way intended to provide an exhaustive consideration of equilibrium dynamics in the presence of indivisibility, but rather illustrates that even a simple approximating model can generate richly varied predictions after accounting for practical frictions. In many ways, this complexity underscores the surprising degree to which excess-demand driven dynamics from an abstract continuous model provide an informative device for predicting price dynamics.

9.3. Alternative Models of Disequilibrium Price Dynamics

9.3.1. Price Dynamics and Equilibrium (Non)-Attraction

In order to evaluate the degree to which equilibrium attraction might shape price dynamics, we consider whether concurrent deviations from equilibrium prices predict future price movements. To test this link, we regress changes in prices on the distance between prices and equilibrium, to which we will refer as the *Equilibrium Attraction Absolute Model*.

$$\begin{bmatrix} \dot{P}_{1,t} \\ \dot{P}_{2,t} \end{bmatrix} \equiv \begin{pmatrix} P_{1,t} - P_{1,t-1} \\ P_{2,t} - P_{2,t-1} \end{pmatrix} = \begin{bmatrix} a_{10} \\ a_{10} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{pmatrix} 40 - P_{1,t-1} \\ 20 - P_{2,t-1} \end{pmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \tag{12}$$

An alternative approach to modeling price movements considers relative, rather than absolute price changes. To evaluate this specification, we regress percentage changes in prices on the distance between prices and equilibrium, by which we define the *Equilibrium Attraction Relative Model*.

$$\begin{bmatrix} \dot{P}_{1,t} / P_{1,t-1} \\ \dot{P}_{2,t} / P_{2,t-1} \end{bmatrix} = \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{pmatrix} 40 - P_{1,t-1} \\ 20 - P_{2,t-1} \end{pmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \tag{13}$$

Table 11 presents the coefficients on prices from estimating the Equilibrium Attraction model for both the absolute (Panel A) and the relative (Panel B) specifications, pooling all sessions into a common treatment.[20] Though a statistically significant relationship, the estimated impact of price deviations from equilibrium on price innovations is nearly zero. To illustrate, for every unit of X_3 that prices deviate from equilibrium prices, we would expect a correction of only 0.022 units in the next transaction.

Table 11: Estimated Coefficients in Equilibrium Attraction Models

	Panel A: Absolute Attraction					*Panel B: Relative Attraction*			
	Estimate	SE	t-Stat	p-val		Estimate	SE	t-Stat	p-val
a_{11}	2.22 E-02	5.38 E-03	4.21	<0.01	a_{11}	2.75 E-04	9.89 E-05	2.75	0.01
a_{12}	-2.28 E-03	5.41 E-03	-0.42	0.67	a_{12}	-4.71 E-05	1.01 E-04	-0.46	0.64
a_{21}	-2.50 E-03	2.41 E-03	-1.03	0.30	a_{21}	-1.72 E-05	7.50 E-05	-2.30	0.02
a_{22}	1.02 E-02	4.62 E-03	2.21	0.03	a_{22}	3.02 E-04	1.45 E-04	2.09	0.04

The Equilibrium Attraction Model, rather than serving as a theoretically grounded model of disequilibrium price dynamics, serve as an econometric specification for testing the degree to which equilibrium prices predict price changes. Indeed, the regression specifications in (12) and (13) can be interpreted as an Error Correction Model where transaction prices follow independent unit root processes converging to the equilibrium prices. Despite lacking a theoretical foundation, this specification provides a viable reduced-form device for testing whether prices' deviation from theoretical equilibrium directly predict price changes. The weakness of this predictive relationship demonstrates the degree to which prices diverge from the theoretical equilibrium.

[20] We apply the same treatment to price changes as we adopt in later sections to estimate the structural relationship between price changes and excess demand. Using price changes at the transaction level, winsorized to limit outlier influence, we estimate all models equation-by-equation using FGLS accounting for Autoregressive Conditional Heteroscedasticity within each session.

9.3.2. Comparing Model Specifications for Price Dynamics

We can combine the regression specifications from the Equilibrium Attraction and Classical Models into an aggregated model that allows us to evaluate the relative explanatory power of Equilibrium Attraction, excess demand in the Classical Model, and excess demand in the Relative Classical Model. The regression equation of the aggregated model for commodity X_1 takes the form:

$$\begin{aligned} P_{1,t} - P_{1,t-1} = a_{10} &+ a_{11}^{EAA}\left(40 - P_{1,t-1}\right) + a_{12}^{EAA}\left(20 - P_{2,t-1}\right) \\ &+ a_{11}^{EAR}\left(40 - P_{1,t-1}\right)P_{1,t-1} + a_{12}^{EAR}\left(20 - P_{2,t-1}\right)P_{1,t-1} \\ &+ a_{11}^{GA} Z_{1,t-1}\left(P_{t-1}\right) + a_{12}^{GA} Z_{2,t-1}\left(P_{t-1}\right) \\ &+ a_{11}^{GR} Z_{1,t-1}\left(P_{t-1}\right)P_{1,t-1} + a_{12}^{GR} Z_{2,t-1}\left(P_{t-1}\right)P_{1,t-1} + \varepsilon_{1,t} \end{aligned} \tag{14}$$

The analogous model for commodity X_2 is constructed similarly. Note that Eq. (14) nests all the models evaluated in the paper, with the parameters superscripted by EAA, EAR, GA, and GR corresponding to the Equilibrium Attraction Absolute, Equilibrium Attraction Relative, Generalized Absolute, and Generalized Relative Models, respectively. Our interest in this specification is purely empirical, as estimating this aggregate regression model allows us to identify which forces are most relevant to explaining price processes. Table 12 presents the regression results for the model in Eq. (14), separately for commodity X_1 (Panel A) and X_2 (Panel B).

Table 12: Estimated Aggregated Dynamic Model Coefficients

Panel A: Commodity X_1 | *Panel B: Commodity X_2*

	Coeff	SE	t-Stat	p-Val		Coeff	SE	t-Stat	p-Val
a_{11}^{EAA}	-3.71 E-02	1.37 E-02	-0.27	0.79	a_{21}^{EAA}	1.27 E-03	5.05E-03	0.25	0.80
a_{12}^{EAA}	-1.50 E-05	7.55 E-03	-0.20	0.84	a_{22}^{EAA}	1.91 E-03	7.20E-03	0.26	0.79
a_{11}^{EAR}	9.03 E-04	4.53 E-05	1.99	0.05	a_{21}^{EAR}	-2.77E-05	1.30E-05	-2.14	0.03
a_{12}^{EAR}	-1.37 E-04	9.09 E-05	-1.50	0.13	a_{22}^{EAR}	3.30 E-05	4.31E-05	0.77	0.44
a_{11}^{GA}	3.05 E-02	2.30 E-02	1.33	0.19	a_{21}^{GA}	9.95 E-03	1.35E-02	0.74	0.46
a_{12}^{GA}	-4.60 E-03	1.49 E-02	-0.31	0.76	a_{22}^{GA}	-2.17 E-03	5.94E-03	-0.37	0.71
a_{11}^{GR}	2.68 E-03	4.79 E-04	5.59	<0.01	a_{21}^{GR}	3.93 E-04	1.77E-04	2.22	0.03
a_{12}^{GR}	4.11 E-04	1.37 E-04	2.99	<0.01	a_{22}^{GR}	2.19 E-03	2.21E-04	9.91	<0.01

We first consider the empirical relevance of the absolute models for characterizing price dynamics, which seem quite limited compared to the relative models. Only one of the eight coefficients associated with an absolute model, a_{21}^{EAA}, achieves marginal significance. However, this significance should be greeted with skepticism given the same coefficient was not statistically significant in the Equilibrium Attraction model specifications presented in Table 11 that did not include excess demand measures in the set of regressors. A Wald test of the joint zero restriction on all eight absolute model coefficients is rejected at the 0.02 significance level, suggesting the absolute measures of excess demand and disequilibrium do have some explanatory power.

We next consider the degree to which equilibrium attraction forces characterize expected price dynamics. The marginal significance results indicate that a_{11}^{EAR} and a_{21}^{EAR} provide statistically significant predictors for expected price dynamics, but their influences are quite small with a price divergence of 100 leading to an expected correction of less than 1%. None of the Absolute Attraction predictors are statistically significant and a joint test that $a_{11}^{EAA} = a_{12}^{EAA} = a_{21}^{EAA} = a_{22}^{EAA} = 0$ is not rejected with a p-Value of 0.45.

Our last observation seeks to evaluate the degree to which partial and general equilibrium adjustments influence expected price dynamics. We begin by noting that the joint restriction $a_{12}^{GA} = a_{21}^{GA} = a_{12}^{GR} = a_{21}^{GR} = 0$ is rejected by the data since a_{12}^{GR} and a_{21}^{GR} both reach the threshold for statistical significance. However, the magnitude of the diagonal coefficients $\left(a_{ii}^{\cdot}\right)$ is clearly much greater than the magnitude of the off diagonal coefficients $\left(a_{ij}^{\cdot}\right)$.

10. APPENDIX 2: Figures Presenting Time Series Results for All Sessions

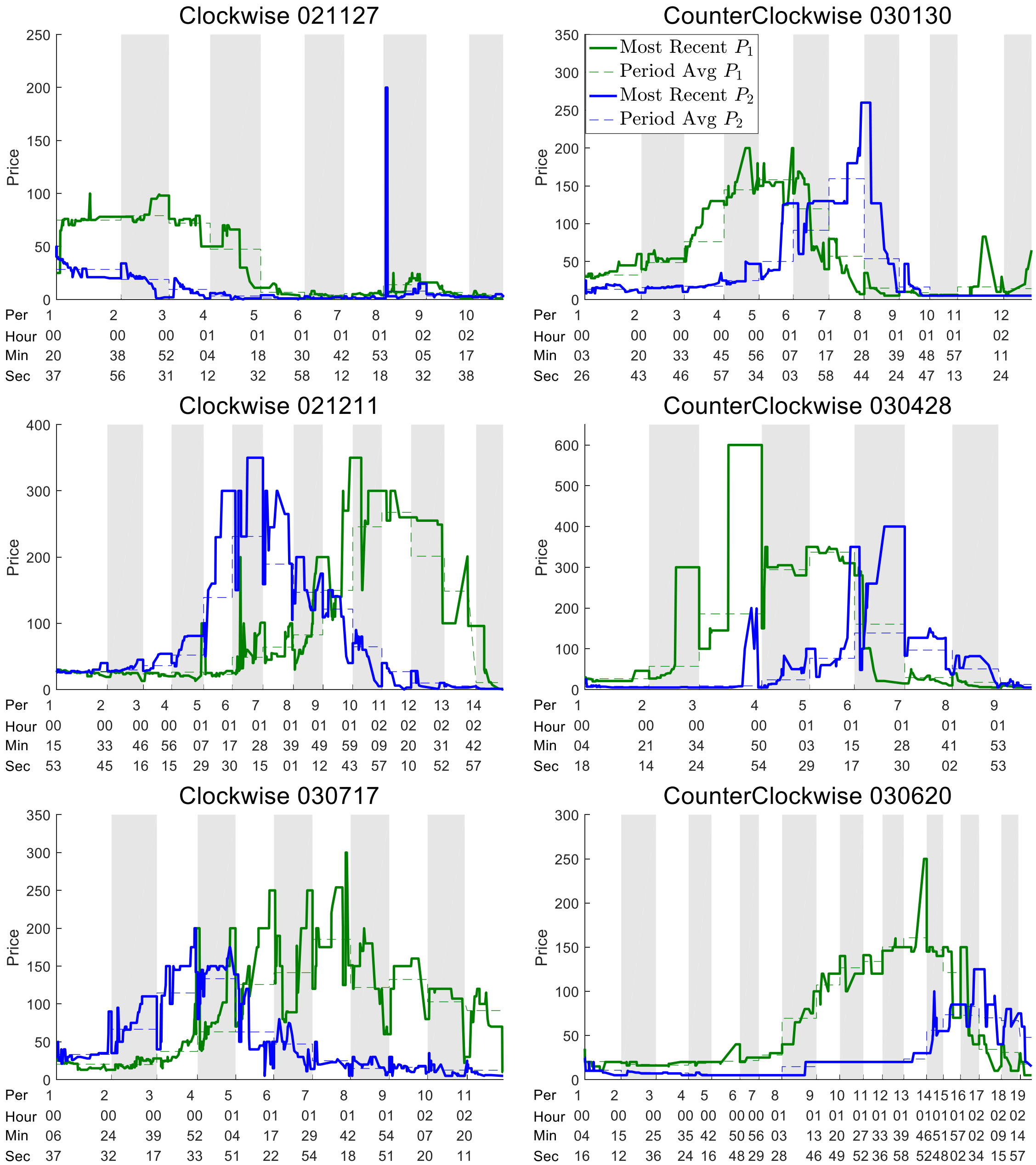

Fig. 16: Transaction Price Time Series (Fig. 3)

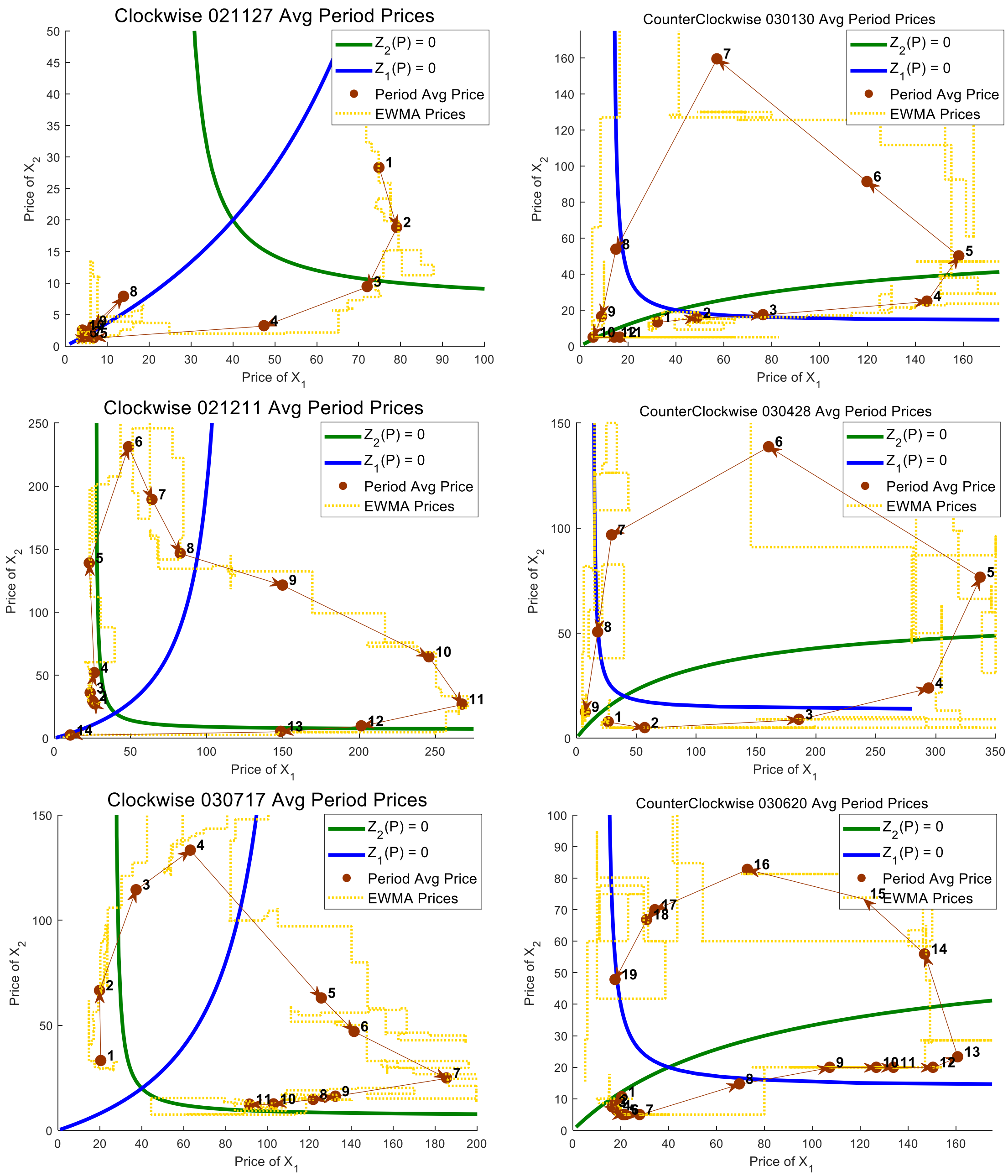

Fig. 17: Period Average Prices and Phase Diagram (Fig. 4)

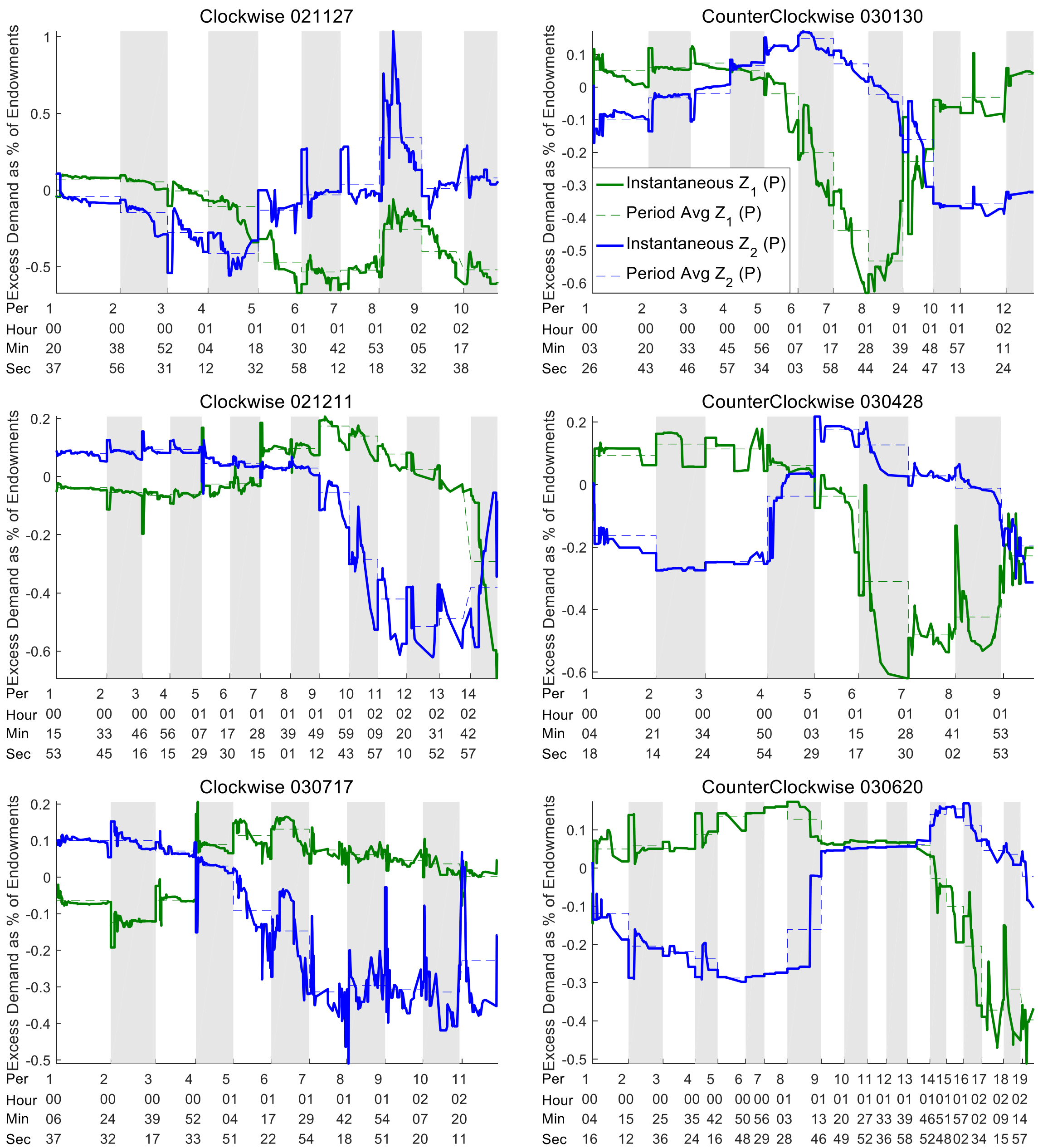

Fig. 18: Excess Demand Dynamics (Fig. 6)

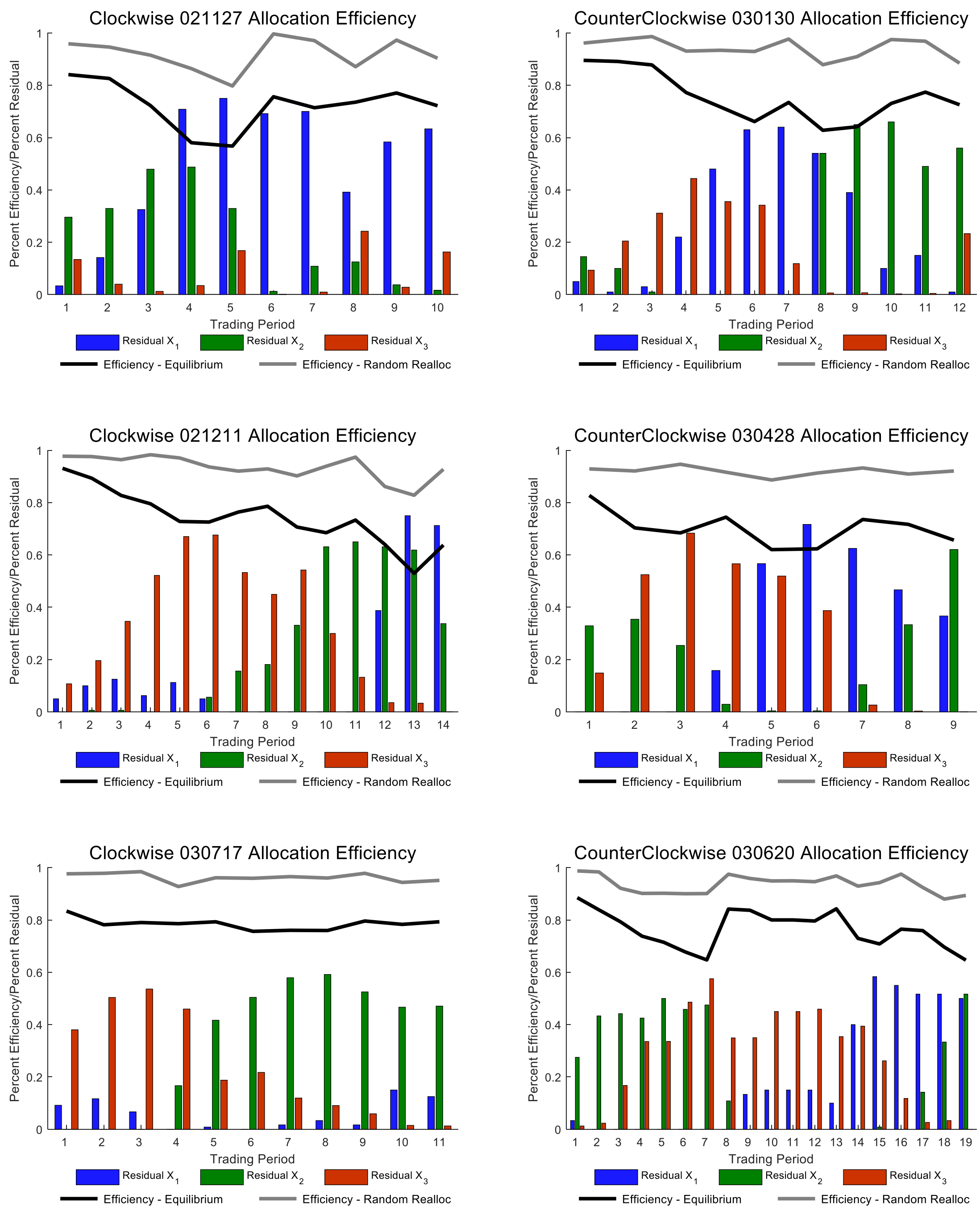

Fig. 19: Period-End Allocation Efficiency (Fig. 8)

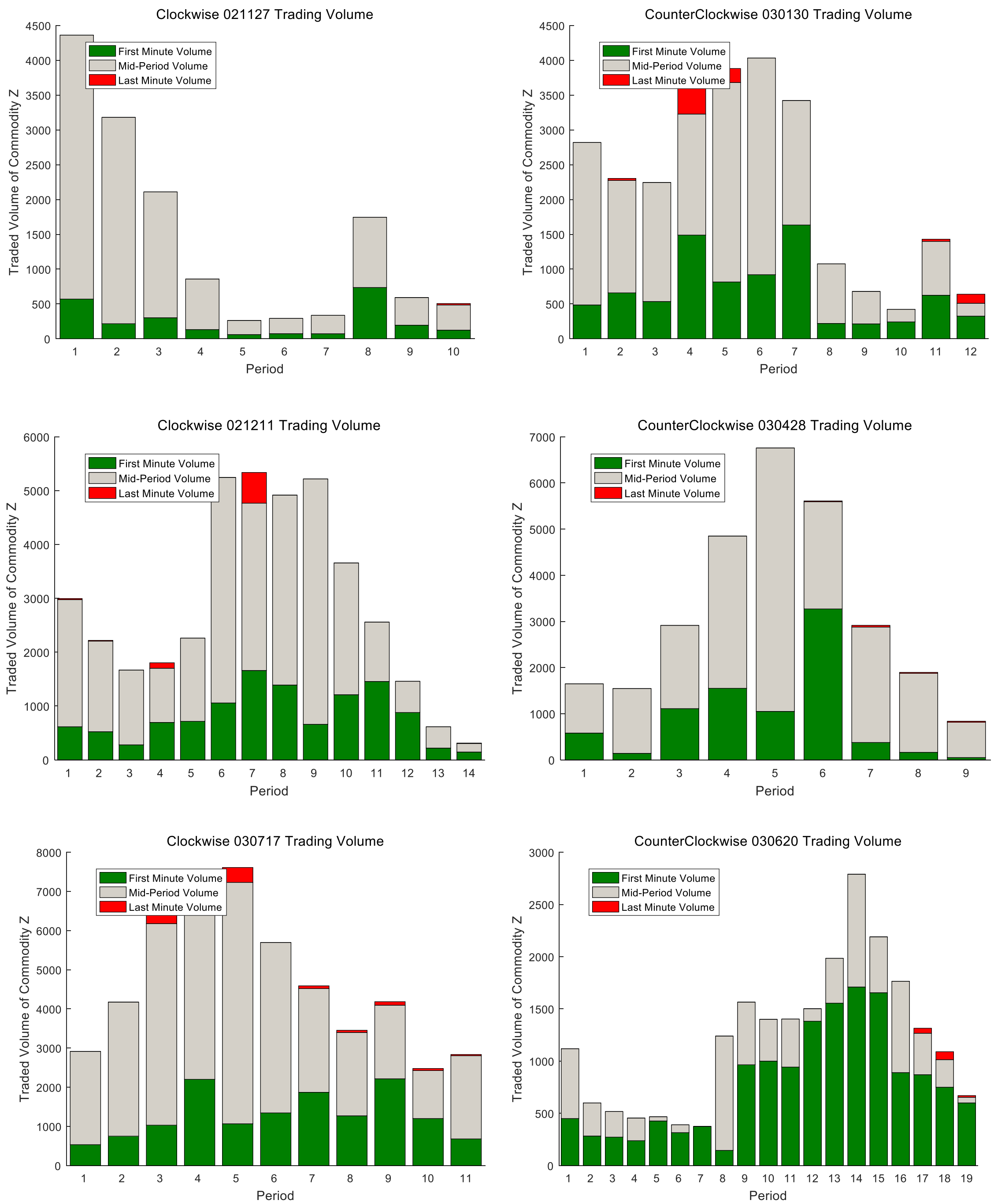

Fig. 20: Early-vs-Late Transaction Volume by Period (Fig. 9)

11. APPENDIX 3: Session-Level Regression Results

Table 13: Session-Level Results for Absolute Equilibrium Attraction Model

	Clockwise - 021127				*Counterclockwise - 030130*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	5.74E-02	-1.92E-01	-4.06E-03	6.37E-02	9.68E-03	2.09E-02	-2.11E-02	7.19E-03
Std Err	1.34E-02	4.33E-02	5.15E-03	8.47E-03	1.12E-02	1.11E-02	8.13E-03	1.12E-02
t-Stat	4.28	-4.44	-0.79	7.52	0.86	1.88	-2.60	0.64

	Clockwise - 021211				*Counterclockwise - 030428*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	3.91E-02	-1.40E-02	9.42E-03	1.32E-02	1.75E-02	1.83E-02	-9.10E-03	2.36E-02
Std Err	1.44E-02	8.92E-03	6.84E-03	9.69E-03	9.79E-03	1.53E-02	4.77E-03	1.26E-02
t-Stat	2.72	-1.57	1.38	1.36	1.79	1.20	-1.91	1.88

	Clockwise - 030717				*Counterclockwise - 030620*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	3.11E-02	-1.46E-03	1.92E-02	1.14E-02	-1.77E-03	8.49E-02	-1.92E-02	2.77E-02
Std Err	1.73E-02	2.01E-02	7.14E-03	1.05E-02	1.26E-02	2.69E-02	1.17E-02	2.38E-02
t-Stat	1.80	-0.07	2.68	1.08	-0.14	3.15	-1.64	1.16

Table 14: Session-Level Results for Relative Equilibrium Attraction Model

	Clockwise - 021127				*Counterclockwise - 030130*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	1.07E-03	-2.71E-03	6.78E-04	2.36E-03	5.18E-04	5.67E-04	-6.89E-04	4.04E-04
Std Err	7.18E-04	2.32E-03	4.29E-04	7.91E-04	3.54E-04	3.49E-04	2.00E-04	2.79E-04
t-Stat	1.49	-1.17	1.58	2.98	1.47	1.62	-3.45	1.45

	Clockwise - 021211				*Counterclockwise - 030428*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	2.58E-04	-3.24E-04	1.55E-04	1.97E-04	1.16E-04	2.94E-04	-3.79E-04	4.40E-04
Std Err	2.24E-04	1.38E-04	2.13E-04	3.02E-04	1.66E-04	2.58E-04	1.17E-04	3.35E-04
t-Stat	1.16	-2.36	0.73	0.65	0.70	1.14	-3.25	1.31

	Clockwise - 030717				*Counterclockwise - 030620*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	2.95E-04	-1.11E-04	2.66E-04	5.52E-04 `	7.69E-05	1.41E-03	-5.91E-04	5.88E-04
Std Err	1.96E-04	2.26E-04	1.98E-04	2.92E-04	2.92E-04	6.25E-04	3.32E-04	6.81E-04
t-Stat	1.51	-0.49	1.34	1.89	0.26	2.25	-1.78	0.86

Table 15: Session-Level Results for Absolute Excess Demand Model

	Clockwise - 021127				*Counterclockwise - 030130*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	2.55E-02	1.25E-03	-1.15E-02	-1.75E-03	-5.08E-02	-6.07E-03	2.11E-02	-3.36E-04
Std Err	9.81E-03	1.42E-03	4.24E-03	5.01E-04	2.33E-02	2.30E-03	1.70E-02	1.76E-03
t-Stat	2.59	0.88	-2.71	-3.50	-2.18	-2.64	1.24	-0.19

	Clockwise - 021211				*Counterclockwise - 030428*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	2.58E-04	-3.24E-04	1.55E-04	1.97E-04	1.16E-04	2.94E-04	-3.79E-04	4.40E-04
Std Err	2.24E-04	1.38E-04	2.13E-04	3.02E-04	1.66E-04	2.58E-04	1.17E-04	3.35E-04
t-Stat	1.16	-2.36	0.73	0.65	0.70	1.14	-3.25	1.31

	Clockwise - 030717				*Counterclockwise - 030620*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	2.95E-04	-1.11E-04	2.66E-04	5.52E-04 `	7.69E-05	1.41E-03	-5.91E-04	5.88E-04
Std Err	1.96E-04	2.26E-04	1.98E-04	2.92E-04	2.92E-04	6.25E-04	3.32E-04	6.81E-04
t-Stat	1.51	-0.49	1.34	1.89	0.26	2.25	-1.78	0.86

Table 16: Session-Level Results for Relative Excess Demand Model

	Clockwise - 021127				*Counterclockwise - 030130*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	8.89E-04	3.88E-05	2.86E-04	-2.98E-05	-3.34E-04	-5.46E-05	2.09E-03	-2.58E-05
Std Err	2.52E-04	1.63E-05	9.03E-04	3.53E-05	5.26E-04	2.84E-05	4.93E-04	2.99E-05
t-Stat	3.53	2.37	0.32	-0.84	-0.63	-1.92	4.24	-0.86

	Clockwise - 021211				*Counterclockwise - 030428*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	9.67E-04	2.06E-08	3.24E-04	-1.17E-05	5.29E-05	1.04E-05	1.69E-03	-1.37E-05
Std Err	2.06E-04	9.49E-06	7.89E-04	6.04E-06	2.58E-04	1.18E-05	3.61E-04	8.56E-06
t-Stat	4.70	0.00	0.41	-1.94	0.21	0.88	4.69	-1.61

	Clockwise - 030717				*Counterclockwise - 030620*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	3.20E-04	-1.03E-05	1.60E-03	-1.39E-05 `	-1.44E-03	-5.20E-05	2.65E-03	-2.84E-04
Std Err	2.60E-04	1.40E-05	7.63E-04	9.87E-06	1.38E-03	6.07E-05	1.86E-03	1.35E-04
t-Stat	1.23	-0.74	2.10	-1.41	-1.04	-0.86	1.42	-2.10

Table 17: Session-Level Results for Aggregated Model

Panel A: Clockwise Treatment Session Level Results

Clockwise - 021127

Good X_1	a_{11}^{EAA}	a_{12}^{EAA}	a_{11}^{EAR}	a_{12}^{EAR}	a_{11}^{GA}	a_{12}^{GA}	a_{11}^{GR}	a_{12}^{GR}
Coeff	-6.12E-02	-9.53E-02	5.42E-03	2.64E-03	-2.32E-03	1.52E-02	1.55E-02	-1.61E-03
Std Err	1.42E-01	1.95E-01	1.44E-03	2.97E-03	7.06E-02	1.52E-02	3.46E-03	6.39E-04
t-Stat	-0.43	-0.49	3.77	0.89	-0.03	1.00	4.50	-2.51
Good X_2	a_{21}^{EAA}	a_{22}^{EAA}	a_{21}^{EAR}	a_{22}^{EAR}	a_{21}^{GA}	a_{22}^{GA}	a_{22}^{GA}	a_{22}^{GR}
Coeff	-1.17E-01	2.77E-01	6.86E-06	-5.16E-03	-2.85E-03	-4.25E-03	-3.31E-03	4.36E-03
Std Err	6.98E-02	1.11E-01	6.56E-04	2.25E-03	3.25E-02	7.40E-03	1.73E-03	1.47E-03
t-Stat	-1.67	2.50	0.01	-2.30	-0.09	-0.58	-1.92	2.96

Clockwise - 021211

Good X_1	a_{11}^{EAA}	a_{12}^{EAA}	a_{11}^{EAR}	a_{12}^{EAR}	a_{11}^{GA}	a_{12}^{GA}	a_{11}^{GR}	a_{12}^{GR}
Coeff	-1.33E-01	-4.18E-02	7.52E-04	7.56E-04	-2.36E-01	8.33E-02	1.22E-02	9.22E-05
SE	9.99E-02	1.74E-02	2.80E-04	3.69E-04	2.10E-01	1.02E-01	2.59E-03	8.62E-04
t-Stat	-1.33	-2.40	2.68	2.05	-1.12	0.82	4.70	0.11
Good X_2	a_{21}^{EAA}	a_{22}^{EAA}	a_{21}^{EAR}	a_{22}^{EAR}	a_{21}^{GA}	a_{22}^{GA}	a_{22}^{GA}	a_{22}^{GR}
Coeff	-8.89E-03	1.20E-02	-3.21E-05	-3.31E-04	1.19E-01	1.35E-02	-4.02E-03	3.30E-03
SE	5.38E-02	1.69E-02	1.38E-04	2.87E-04	1.85E-01	5.35E-02	1.75E-03	1.23E-03
t-Stat	-0.17	0.71	-0.23	-1.15	0.65	0.25	-2.30	2.68

Clockwise - 030717

Good X_1	a_{11}^{EAA}	a_{12}^{EAA}	a_{11}^{EAR}	a_{12}^{EAR}	a_{11}^{GA}	a_{12}^{GA}	a_{11}^{GR}	a_{12}^{GR}
Coeff	-2.20E-01	-5.80E-02	7.58E-04	1.49E-03	-6.41E-02	-3.34E-02	4.29E-03	1.44E-03
SE	1.16E-01	4.01E-02	4.31E-04	7.43E-04	2.53E-01	1.01E-01	3.47E-03	1.04E-03
t-Stat	-1.89	-1.45	1.76	2.00	-0.25	-0.33	1.24	1.39
Good X_2	a_{21}^{EAA}	a_{22}^{EAA}	a_{21}^{EAR}	a_{22}^{EAR}	a_{21}^{GA}	a_{22}^{GA}	a_{22}^{GA}	a_{22}^{GR}
Coeff	-8.41E-02	-3.12E-02	1.71E-04	6.19E-04	-1.36E-01	2.93E-02	1.68E-03	3.17E-03
SE	6.50E-02	3.57E-02	2.21E-04	3.92E-04	1.91E-01	3.95E-02	1.88E-03	1.09E-03
t-Stat	-1.29	-0.88	0.77	1.58	-0.71	0.74	0.89	2.92

Table 18: Session-Level Results for Aggregated Model (cont.)

Panel B: Counterclockwise Treatment Session Level Results

Counterclockwise - 030130

Good X_1	a_{11}^{EAA}	a_{12}^{EAA}	a_{11}^{EAR}	a_{12}^{EAR}	a_{11}^{GA}	a_{12}^{GA}	a_{11}^{GR}	a_{12}^{GR}
Coeff	-3.31E-01	3.67E-02	2.56E-03	-1.21E-03	-2.15E-01	-1.46E-01	7.65E-03	3.29E-03
Std Err	9.61E-02	3.85E-02	6.23E-04	7.23E-04	1.12E-01	6.37E-02	2.56E-03	1.02E-03
t-Stat	-3.44	0.95	4.11	-1.68	-1.92	-2.29	2.99	3.23
Good X_2	a_{21}^{EAA}	a_{22}^{EAA}	a_{21}^{EAR}	a_{22}^{EAR}	a_{21}^{GA}	a_{22}^{GA}	a_{22}^{GA}	a_{22}^{GR}
Coeff	1.85E-02	2.82E-01	-8.02E-05	4.57E-04	2.65E-02	3.47E-02	-3.53E-03	9.58E-03
Std Err	4.85E-02	1.41E-01	2.69E-04	7.43E-04	4.81E-02	3.82E-02	2.02E-03	1.96E-03
t-Stat	0.38	2.00	-0.30	0.62	0.55	0.91	-1.75	4.90

Counterclockwise - 030421

Good X_1	a_{11}^{EAA}	a_{12}^{EAA}	a_{11}^{EAR}	a_{12}^{EAR}	a_{11}^{GA}	a_{12}^{GA}	a_{11}^{GR}	a_{12}^{GR}
Coeff	-6.92E-02	-7.08E-03	2.03E-04	-8.04E-04	-1.05E-01	-9.32E-02	7.10E-03	5.82E-04
Std Err	4.41E-02	3.04E-02	1.04E-04	2.75E-04	9.39E-02	1.10E-01	2.01E-03	3.89E-04
t-Stat	-1.57	-0.23	1.95	-2.92	-1.12	-0.84	3.53	1.50
Good X_2	a_{21}^{EAA}	a_{22}^{EAA}	a_{21}^{EAR}	a_{22}^{EAR}	a_{21}^{GA}	a_{22}^{GA}	a_{22}^{GA}	a_{22}^{GR}
Coeff	1.63E-02	-1.98E-01	-4.00E-05	4.24E-04	1.10E-02	-4.47E-03	3.04E-03	1.79E-03
Std Err	1.23E-02	9.16E-02	2.45E-05	1.86E-04	3.98E-02	3.39E-02	1.21E-03	4.62E-04
t-Stat	1.32	-2.16	-1.64	2.27	0.28	-0.13	2.51	3.87

Counterclockwise - 030620

Good X_1	a_{11}^{EAA}	a_{12}^{EAA}	a_{11}^{EAR}	a_{12}^{EAR}	a_{11}^{GA}	a_{12}^{GA}	a_{11}^{GR}	a_{12}^{GR}
Coeff	-1.68E-01	-1.38E-01	1.69E-03	1.54E-03	1.48E-01	-1.02E-01	1.14E-02	5.49E-03
Std Err	9.60E-02	1.69E-01	5.31E-04	2.61E-03	4.09E-01	2.12E-01	9.89E-03	3.51E-03
t-Stat	-1.75	-0.82	3.18	0.59	0.36	-0.48	1.15	1.57
Good X_2	a_{21}^{EAA}	a_{22}^{EAA}	a_{21}^{EAR}	a_{22}^{EAR}	a_{21}^{GA}	a_{22}^{GA}	a_{22}^{GA}	a_{22}^{GR}
Coeff	1.40E-01	-6.42E-02	-5.07E-04	7.92E-03	-6.65E-01	-1.27E-01	1.57E-02	5.72E-02
Std Err	8.23E-02	3.19E-01	5.71E-04	1.58E-03	2.49E-01	1.87E-01	9.31E-03	7.17E-03
t-Stat	1.70	-0.20	-0.89	5.02	-2.67	-0.68	1.69	7.97

REFERENCES

Anderson, C.M., Plott, C.R., Shimomura, K.I., Granat, S.: Global instability in experimental general equilibrium: the Scarf example. J. Econ. Theory 115(2), 209-249 (2004). https://doi.org/10.1016/S0022-0531(03)00185-6

Crockett, S., Oprea, R., Plott, C.R.: Extreme Walrasian dynamics: the Gale example. Am. Econ. Rev. 101, 3196–3220 (2010)

Donzelli, F.: Edgeworth versus Walras on equilibrium and disequilibrium. In: UNIMI—Research Papers in Economics, Business, and Statistics, unimi-1096, Universitá degli Studi di Milano (2009)

Gintis, H.: The dynamics of general equilibrium. Econ. J. 117(523), 1280–1309 (2007)

Goeree, J., Lindsey, L.: Market design and the stability of general equilibrium. J. Econ. Theory 165, 37–68 (2016). https://doi.org/10.1016/j.jet.2016.04.004

Hirota, M.: On the stability of competitive equilibrium and the patterns of initial holdings : an example. Int. Econ. Rev. 22, 461–467 (1981)

Kiyotaki, N., Wright, R.: On money as a medium of exchange. J. Polit. Econ. 97(4), 927–954 (1989)

McFadden, D.: On Hicksian stability. In: Wolfe, J.N. (ed.) Value, Capital and Growth, pp. 329–351. University Press, Edinburgh (1969)

McKenzie, L.W.: Classical General Equilibrium Theory. MIT Press, CAMBRIDGE (2002)

Mukherji, A.: A locally stable adjustment process. Econometrica 63(2), 441–448 (1995)

Mukherji, A.: An Introduction to General Equilibrium Analysis. Oxford University Press, Chennai (2002)

Mukherji, A.: Competitive Equilibria: Convergence, Cycles or Chaos, Centre for Economic Studies and Planning. Jawaharlal Nehru University, New Delhi (2003)

Mukherji, A.: A global stability condition: generalized law of demand. In: Presented at: International Conference on Economic Theory, Tokyo, March 2004, Organized by Keio University and Kyoto University (2004)

Mukherji, A.: The second fundamental theorem of positive economics. Int. J. Econ. Theory 8, 125–138 (2012). https://doi.org/10.1111/j.1742-7363.2012.00181.x

Mukherji, A.: General equilibrium: a status report. In: Sinha, A and Thomas, A.M. (eds), Pluralistic Economics and Its History, Routledge N.Y., 191-209 (2019)

Mukherji, A., Guha, S.: Mathematical Methods and Economic Theory. Oxford University Press, New Delhi (2011)

Plott, C.R.: Market stability: backward bending supply in a laboratory market. Econ. Inq. 38(1), 1–18 (2000)

Plott, C.R.: Equilibrium, equilibration, information and multiple markets: from basic scie nce to institutional design. In: Nobel Symposium Behavioral and Experimental Economic s (2001)

Plott, C.R., George, G.: Marshallian vs. Walrasian stability in an experimental market. Ec on. J. 102, 437–460 (1992)

Plott, C.R., Gray, P.: The multiple unit double auction. J. Econ. Behav. Organ. 13, 245–258 (1990)

Plott, C.R., Pogorelskiy, K.: Call market experiments: efficiency and price discovery thro ugh multiple calls and emergent newton adjustments. Am. Econ. J. Microecon. 9(4), 1–41 (2017)

Plott, C.R., Smith, J.: Instability of equilibria in experimental markets: upward-sloping demands, externalities, and fad-like incentives. South. Econ. J. 65(3), 405–426 (1999)

Plott, C., Smith, V.: An experimental examination of two exchange institutions. Rev. Eco n. Stud., 45(1), 133–153 (1978). https://EconPapers.repec.org/RePEc:oup:restud:v:45:y:1978:i:1:p:133-153

Plott, C.R., Smith, V.L. (eds.): Handbook of Experimental Economics Results, vol. 1. North-Holland, Amsterdam (2008)

Scarf, H.: Some examples of global instability of the competitive equilibrium. Int. Econ. Rev. 1, 157–171 (1960)

Shen, J., Shimomura, K.I., Yamato, T., Ohtaka, T., Takahashi, K.: Revisiting Marshallian versus Walrasian stability in an experimental market. In: Kawano, M., Higano, Y. (eds.), Theory and History in Regional Perspective: In Honor of Professor Yasuhiro Sakai. Spri nger, Berlin (forthcoming)

Smith, V.L.: Experimental auction markets and the Walrasian hypothesis. J. Polit. Econ. 73(4), 387–393 (1965)

Walker, D.A.: Edgeworth versus Walras on the theory of Tatonnement. East. Econ. J. 13(2), 155–165 (1987)

Walras, L.: Elements of Pure Economics (William Jaffé, Trans.). London: Routledge (original work published 1899) (1954)